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Difference-Form Persuasion Contests

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Abstract

We explore the equilibrium properties of two types of "difference-form" persuasion contest functions derived in Skaperdas and Vaidya in which contestants spend resources to persuade an audience. We find that both types of functions generate interior pure strategy Nash equilibria unlike Baik and Che and Gale with characteristics different to existing literature. For one type of function, we find that the reaction function of each player is "flat" and nonresponsive to the level of resources devoted by the rival so that the "preemption effect" as defined by Che and Gale is absent. Further, the equilibrium is invariant to the sequencing of moves. For the second type of function, which applies when there is asymmetry among contestants with regard to the quality of evidence, we find that the reaction functions of the stronger and weaker players have gradients with opposite signs relative to Dixit and therefore their incentive to precommit expenditures in a sequential move game is also different. For both types of functions, the extent of rent dissipation is partial. From the equilibrium analysis, we are also able to establish the potential effects of some specific factors affecting persuasion such as evidence potency, the degree of truth, and bias on aggregate resource expenditures and welfare.

1. Introduction

A large variety of economic activities can be thought to be about persuasion where competing parties attempt to influence the opinions and hence the decisions of their

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relevant audiences through costly production of "information" or evidence.[1] These include, among many others, advertising (Schmalensee 1972), electoral campaigning (Snyder 1989; Baron 1994; Skaperdas and Grofman 1995), marketing (Bell, Keeney, and Little 1975), litigation (Farmer and Pecorino 1999; Bernardo, Talley, and Welch 2000; Hirshleifer and Osborne 2001; Robson and Skaperdas 2008), and rent-seeking or lobbying (Tullock 1980). In each of these settings, contest functions have often been employed to translate the resources or costly efforts employed by the competing parties into probabilities of their view prevailing over the relevant audience.[2]

However, until recently the persuasion process by which resources expended by the contestants translate into the win probabilities governed by such functions has not been clarified. In the lobbying context, resources expended by competing sides to influence a decision maker are often considered venal—where they are interpreted as transfers or bribes.[3] However, such an interpretation does not encompass lobbying activities that can be naturally thought of as persuasion even when no bribes are exchanged.

Skaperdas and Vaidya (2012) explicitly derive the contest functions as win probabilities in a game of persuasion where competing parties invest resources to produce evidence from which an audience updates its priors using Bayesian inference. They show that both ratio-form and difference-form contest functions can be derived as an outcome of such a process. In this paper, we examine the equilibrium characteristics of both the symmetric and asymmetric versions of the difference-form contest function derived in their paper as reproduced in (1) and (2), respectively, as follows:[4]

$$p_1(R_1, R_2) = \frac{1}{2} + \frac{\alpha}{2}\left[h(R_1) - h(R_2)\right], \tag{1}$$

$$\begin{aligned} p_1(R_1, R_2) = (1-\gamma) + \gamma \Bigg\{ &\left[\left(\frac{\Gamma - 1}{\Gamma}\right) h(R_1) - \left(\frac{1-\delta}{\delta}\right) h(R_2)\right] \\ &+ \left[\left(\frac{1-\delta}{\delta}\right) - \left(\frac{\Gamma-1}{\Gamma}\right)\right] h(R_1) h(R_2) \Bigg\}. \end{aligned} \tag{2}$$

The form in (2) is more general relative to (1), where $h(.)$ represents the probability of a player finding favorable evidence, $0 < \gamma < 1$ represents the audience's decision threshold, and $\frac{\Gamma-1}{\Gamma}$ and $\frac{1-\delta}{\delta}$ represent the evidence potencies of the competing players. Biases can arise when $\gamma \neq \frac{1}{2}$ so that the bar is relatively higher for one of the parties or when there are differences in evidence potencies. Under symmetry, when evidence potencies are identical and $\gamma = \frac{1}{2}$, (2) naturally reduces to the form in (1).

Since (1) and (2) are explicitly grounded in the persuasion context, the parameters in the functions have natural inferential interpretations, thus making them particularly suitable to contests aimed at persuading a relevant audience such as marketing, adver-

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[1] For a recent survey on the quantitative impact of such persuasion activities in voting, marketing, and financial markets, see DellaVigna and Gentzkow (2010).

[2] See Corchon (2007) and Jia, Skaperdas, and Vaidya (2013) for an overview of the theoretical foundations and applications of contest functions.

[3] See, for example, Grossman and Helpman (1994).

[4] Tullock (1980) originally applied a ratio-form contest in the context of rent-seeking. Subsequently, the equilibrium characteristics of the ratio-form contest have already been extensively examined in the contest literature and are well known. See Perez-Castrillo and Verdier (1992) and Nitzan (1994).

tising, electoral campaigning, litigation, and lobbying.[5] These functional forms are also of interest because their characteristics are different to the other general difference-form contests that have been studied in the literature by Lazear and Rosen (1981), Baik (1998), and Che and Gale (2000).[6] In particular, Lazear and Rosen (1981) study incentive provision via a rank-order tournament by employing a symmetric difference-form contest of the form $G(R_1 - R_2)$, which is a twice-continuously differentiable function of the differences in the resource expenditures of the rivals. Baik (1998) studies two-player difference-form contests in which the win probability of player 1 takes the form[7]

$$\begin{aligned} p_1(R_1, R_2) = f(d), \ \text{where } d = \sigma R_1 - R_2, \sigma > 0, \ f' > 0, \\ f'' < 0 \text{ for } d > 0 \text{ and } f(-d) = 1 - f(d). \end{aligned} \tag{3}$$

Che and Gale (2000) examine two-player contests involving the piecewise linear difference-form contest function as given by[8]

$$p_1(R_1, R_2) = \max\left\{\min\left\{\frac{1}{2} + s(R_1 - R_2), 1\right\}, 0\right\}, \text{where } s > 0. \tag{4}$$

Qualitatively, (1) is different from the piecewise linear difference-form contest examined by Che and Gale (2000) as specified in (4) due to the nonlinearity induced by the evidence realization probability function $h(.)$, which by assumption is bounded between 0 and 1, strictly increasing and strictly concave in the resources expended by each party, R_i, $i = 1, 2$. Also, (1) is different from the difference-form functions examined by Lazear and Rosen (1981) and Baik (1998) (as in (3)) because it represents an additive concave transformation of the differences in resources.

The form in (2) is further apart from (3) and (4) due to the presence of a cross-product term $h(R_1)h(R_2)$. To gain better intuition, the form in (2) can also be rearranged to $p_1(R_1, R_2) = (1-\gamma) + \gamma[(\frac{\Gamma-1}{\Gamma})h(R_1)(1-h(R_2)) - (\frac{1-\delta}{\delta})h(R_2)(1-h(R_1))]$. In this representation, the product $h(R_i)(1-h(R_j))$ $i, j = 1, 2, i \neq j$ represents the *ex ante* probability of the event where only player i gets the evidence and thus gains from shifting the audience's posterior belief in its favor. The form in (2) thus naturally attaches a higher weight to this event for the player with the stronger evidence. Hence, in an otherwise symmetric situation where $\gamma = \frac{1}{2}$ and $R_1 = R_2 = R$,

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[5] As discussed by Bevia and Corchon (2015), in some persuasion contexts such as voting and marketing a fraction of the audience may exhibit loyalty to one of the contenders regardless of the expenditures incurred by either contestant. Such behavior is hard to reconcile with more widely used ratio-form Tullock contests where the win probability is either 0 or 1 depending on whether a player chooses not to invest any resources or invest infinitely large amount of resources. The difference-form functions given by (1) and (2) can accommodate such behavior because the win probabilities are bounded away from 0 regardless of the level of resources invested by the competing players.

[6] Hirshleifer (1989) was among the first to explore the equilibrium characteristics of a logistic difference-form contest and showed that they can be considerably different from a Tullock contest. For applications of such logistic form contest to rent-seeking, see Munster and Staal (2011, 2012). See Hwang (2012) for an exploration of a more generalized version of the logistic form contest.

[7] The win probability of player 2 is always $1 - p_1(R_1, R_2)$. Hirshleifer (1989) can be understood as a special case of (3).

[8] Pelosse (2014) and Polishchuk and Tonis (2013) illustrate specific conditions under which a difference-form contest function of the type in (4) can arise as an optimal allocation mechanism for the prize allocator in a rent-seeking context. Corchon and Dahm (2010, 2011) provide alternative positive and normative foundations for this contest function and also extend it to a three-player setting. Their formulation allows for nonlinearity of the type R_i^σ, where $\sigma > 0$. Grossman and Helpman (1996) apply this function to determine the effect of campaign contributions on voting behavior of "uninformed" voters.

$p_1(R_1, R_2)$ reduces to $p_1(R, R) = \frac{1}{2} + \frac{1}{2}h(R)(1 - h(R))[(\frac{\Gamma-1}{\Gamma}) - (\frac{1-\delta}{\delta})]$. Intuitively, in such a situation, a player has an advantage ($p_1(R, R) > \frac{1}{2}$) only if she has stronger evidence ($(\frac{\Gamma-1}{\Gamma}) > (\frac{1-\delta}{\delta})$).[9]

Our analysis shows that (1) and (2) are able to support both corner and strictly interior pure strategy equilibria with characteristics that are different to existing literature.[10] In the case of (3) (and some of its asymmetric variants), we find that the reaction function of each contestant is "flat," that is, independent of the level of resources devoted by the rival as the contest induced by the additive form (1) is inherently nonstrategic. Hence, the equilibria of the simultaneous and sequential move games are identical and involve dominant strategies. Further, the "preemption effect" as defined by Che and Gale (2000) is absent. Hence, an increase in the prize of the higher stake player does not reduce aggregate resource spending.[11] In the case of (2), which applies when contestants differ with regards to the quality of evidence, the reaction functions of the stronger and weaker players have gradients with opposite signs relative to Dixit (1987). Hence under (2), whereas the stronger player reduces her resource investment if the rival expends more, the opposite is true for the weaker player because the latter has a greater incentive to invest resources at the margin. Due to this, the players' precommitment incentives in a sequential move contest involving (2) can be different to those identified in Dixit (1987).[12] Further, in contrast to the Tullock contest, we find that increasing asymmetry among players can lead to higher aggregate resource expenditures in some circumstances.

By studying the equilibrium characteristics of these contest functions, we are also able to establish the distinct impacts of specific persuasion parameters on equilibrium level of resources and win probabilities. We find that a symmetric increase in the quality of evidence available to each side has the potential of intensifying resource expenditures into the contest under some conditions. When evidence qualities differ, so that there is evidence bias in favor of one player, it is possible for the weaker player to have a higher marginal incentive to invest resources relative to the stronger player in an attempt to offset the bias. Threshold bias which leads to one side facing a higher bar to prove its case may not lead to asymmetry in resource expenditures but affects the level of rent

[9] It is worth noting, however, that similar to (3) and (4), (1) and (2) do not exhibit the homogeneity of degree zero property, which is satisfied by the difference-forms examined by Alcalde and Dahm (2007) and Bevia and Corchon (2015). See Skaperdas (1996) for more discussion of the implications of the homogeneity property.

[10] The existence of interior pure strategy Nash equilibria where both parties contribute positive resources in equilibrium is also found in Lazear and Rosen (1981), Alcalde and Dahm (2007), and Bevia and Corchon (2015) but not in Baik (1998) and Che and Gale (2000).

[11] Similar to Alcalde and Dahm (2007), the way preemption effect is defined matters in our set up. Che and Gale (2000) define it as the negative impact on aggregate resource expenditures of an increase in the prize of the higher stake player. By this definition, there is no preemption effect under (1). In Alcalde and Dahm (2007), preemption effect as defined by Che and Gale (2000) exists for a range of values of the scale parameter of the serial contest. Alcalde and Dahm (2007) also provide an alternative definition of preemption effect, which is a decrease in aggregate resource expenditure due to a decrease in the prize of the lower stake player. When defined this way, preemption effect exists in contests defined by (1) and Alcalde and Dahm (2007). However, unlike the latter where both players reduce their spending, in our case only the weaker player reduces her spending. Preemption effect always exists in Che and Gale (2000) using either definition.

[12] Bevia and Corchon (2015) use a symmetric relative difference contest success function with asymmetric stakes to find that players' precommitment incentives are identical to those of Dixit (1987).

dissipation.[13] We also find that when production of evidence favors the side with the truth, equilibrium choices may reinforce the initial advantage to the truthful side.

The paper is organized as follows. Section 2 provides a brief introduction to the difference-form contest functions of the type (1) and (2) as derived in Skaperdas and Vaidya (2012), hereafter referred to as "persuasion functions." Section 3 examines equilibria of contests involving the symmetric persuasion function as given by (1). Section 4 examines equilibrium behavior involving asymmetric versions of (1) as well as those involving (2). In all such cases, to isolate the effect of the specific asymmetry introduced, all other aspects of the game are left symmetric for the two contestants. In both these sections, we also explore the impact of changes in various persuasion parameters on aggregate resource spending and welfare. Section 5 concludes.

2. An Introduction to Difference-Form Persuasion Functions

In this section, we briefly review the difference-form persuasion function as derived by Skaperdas and Vaidya (2012) as an outcome of a stochastic evidence production process. In their setting, two players (denoted by subscript $i = 1, 2$) compete to gather and present evidence in order to influence the verdict of a third-party audience in their favor. Each player i can either produce a discrete piece of evidence in her favor denoted by e_i, or offer no evidence, denoted by e_φ. The production of such evidence is stochastic so that the amount of resources devoted by player i as denoted by R_i enhances her probability of finding favorable evidence $h_i(R_i)$.[14] It is assumed that $0 < h_i(R_i) < 1$, $h_i'(R_i) > 0$, and $h_i''(R_i) < 0$. Thus, depending on evidence realization, there are four possible states of the world that can be observed by the third-party audience: (e_1, e_2), (e_1, e_φ), (e_φ, e_2), and (e_φ, e_φ) occurring with the following probabilities: $h_1(R_1)h_2(R_2)$, $h_1(R_1)[1 - h_2(R_2)]$, $[1 - h_1(R_1)]h_2(R_2)$, and $[1 - h_1(R_1)][1 - h_2(R_2)]$, respectively. Each of these alternative states of the world can induce the audience to revise its prior probability of Player 1 being the "correct" side denoted as $\pi\,(0 < \pi < 1)$ with a posterior $\pi^*(e_i, e_j)$, where $i = 1, \varphi$ and $j = 2, \varphi$. Skaperdas and Vaidya (2012) employ the following parameterization:[15]

$$\pi^*(e_\varphi, e_\varphi) = \pi^*(e_1, e_2) = \pi;$$

$$\pi^*(e_\varphi, e_2) = \delta\pi \text{ for some } \delta \in (0, 1);$$

[13] Let the prize of each player i be denoted as v_i, $i = 1, 2$. Throughout the paper, rent dissipation is defined as $\frac{\sum_{i=1}^{2} R_i}{\mathrm{Max}\{v_1, v_2\}}$. Hence, rent dissipation is considered partial when $\sum_{i=1}^{2} R_i < \mathrm{Max}\{v_1, v_2\}$.

[14] As an example, consider the Obama campaign's investment of resources of over $3 million into collecting and studying information about potential supporters as gleaned from Facebook, voter logs, and telephone and in-person conversations with an objective to deliver personalized messages that are most likely to be effective in mobilizing potential voters. For details see "Obama Mines for Voters with High-Tech tools," *New York Times*, March 8, 2012. Alternatively, one could also view the "evidence" as winning an endorsement from an entity considered as credible by the decision maker. One could also view this process as shopping for the right delegate to represent their case where the quality of the delegate has a bearing on how the case is represented and therefore on how the decision maker rules.

[15] Note that the posterior probability of the audience responds purely to the evidence in front of it and does not take into account the strategies of the competing parties in terms of the resources they put into the contest. This follows from the "limited-world" Bayesian assumption about the audience in Skaperdas and Vaidya (2012). For some empirical and experimental evidence on such nonstrategic inference by specific audiences see DellaVigna and Gentzkow (2010), Malmendier and Shanthikumar (2007), De Franco, Lu, and Vasvari (2007), and Cain, Loewenstein, and Moore (2005). Eyster and Rabin (2010) examine a model where the receivers of information adjust too little for sender's credibility.

$$\pi^*(e_1, e_\varphi) = \begin{cases} \Gamma\pi & \textit{if } \Gamma \le 1/\pi \\ 1 & \textit{if } \Gamma > 1/\pi \end{cases} \qquad \textit{where } \Gamma > 1.$$

Assuming that (i) the audience employs a threshold rule and decides in favor of Player 1 iff $\pi^*(e_i, e_j) > \gamma\,(0 < \gamma < 1)$, (ii) γ is common knowledge, and (iii) the two players do not observe π but have a common uniform prior over it, it is shown that when $\delta > \gamma$, the win probability of player 1 takes the following difference-form:[16]

$$p_1(R_1, R_2) = (1-\gamma) + \gamma\left\{\left[\left(\frac{\Gamma-1}{\Gamma}\right)h_1(R_1) - \left(\frac{1-\delta}{\delta}\right)h_2(R_2)\right] + \left[\left(\frac{1-\delta}{\delta}\right) - \left(\frac{\Gamma-1}{\Gamma}\right)\right]h_1(R_1)h_2(R_2)\right\}. \tag{5}$$

Player 2's win probability is given by $p_2(R_1, R_2) = 1 - p_1(R_1, R_2)$.

The above discussed discrete evidence setting bears resemblance to Dewatripont and Tirole (1999), who examine a principal's problem of incentivizing agents to collect evidence on competing causes via decision-based rewards. They identify reward or "prize" structures under which a contest between advocates of two competing causes produces relevant evidence at a lower cost compared to a single agent collecting evidence for both the causes. This is different from Skaperdas and Vaidya (2012), whose focus is positive characterization of a persuasion contest. Accordingly, in their setting the prize structure is exogenous and the model considers dichotomous outcomes where either Player 1 or 2 always win. Such positive characterization allows the parameters of $p_1(R_1, R_2)$ to have natural inferential interpretations.

As briefly discussed in the introduction, the level of γ captures potential bias in decision threshold level used by the audience. When $\gamma < \frac{1}{2}$, there is bias in favor of Player 1 as the bar for posterior probability is lowered and the audience is more easily convinced about Player 1's position and vice versa. $(\frac{\Gamma-1}{\Gamma})$ represents the "evidence potency" or the inferential power of Player 1's evidence (e_1). Note that, intuitively, it increases in Γ (the factor by which the audience's prior is revised in favor of Player 1). Similarly, $(\frac{1-\delta}{\delta})$ represents Player 2's evidence potency and intuitively, it declines with δ. Note that these parameters allow for various asymmetries so that $p_1(R_1, R_2) \neq p_2(R_1, R_2)$ when $R_1 = R_2$. The sources of such asymmetries include a bias in the threshold $(\gamma \neq \frac{1}{2})$, or differences in evidence potency as captured via $(\frac{\Gamma-1}{\Gamma})$ and $(\frac{1-\delta}{\delta})$ as well as differences in the evidence production functions as embodied in $h_i(R_i)$. The implications of these asymmetries for equilibrium behavior of the contestants are explored in detail in Section 4.

When $h_i(.) = h_j(.) = h(.)$, (5) simplifies to (2). In addition, when $\gamma = \frac{1}{2}$ and $(\frac{\Gamma-1}{\Gamma}) = (\frac{1-\delta}{\delta}) = \alpha$, (5) simplifies further to the symmetric form in (1).

Denoting player i's valuation of the prize as $v_i > 0$, the expected payoff to player i, $i = 1, 2$ is given by

$$U^i(R_i, R_j) = p_i(R_i, R_j)v_i - R_i \text{ for } i, j = 1, 2 \text{ and } i \neq j. \tag{6}$$

[16] When $\delta \le \gamma$, Player 1's win probability from the (e_φ, e_2) state is 0. This leads to δ term dropping out from (5) and therefore some changes to the coefficients for $h_2(R_2)$ and $h_1(R_1)h_2(R_2)$. However, the analytical form of the persuasion function and the presence of the cross-product term persists as per (5). Hence for brevity, we only consider the $\delta > \gamma$ case in the paper. A derivation of (5) can be found in Skaperdas and Vaidya (2012).

In the above expression, let $p_i(R_i, R_j)$ be given by (1) or (2). In the paper, we will examine simultaneous move contests where player i chooses R_i to maximize U^i taking R_j as given for $i, j = 1, 2$ and $i \neq j$ except when we specifically alter the timing of moves. Note that for $i, j = 1, 2$ and $i \neq j$, since $0 < p_i(R_i, R_j) < 1$, it follows that $U^i > 0$ at $R_i = 0$ for any R_j. Further, $U^i < 0$ at $R_i = v_i$ for any R_j. Given this, the strategy space for each player can be restricted to the interval $R_i \in [0, \max\{v_1, v_2\}]$, $i = 1, 2$ without loss of generality. Throughout the paper we also assume strict concavity of $h(.)$ over $R_i \in [0, \max\{v_1, v_2\}]$ to facilitate sufficient conditions for interior equilibria. These conditions are stated explicitly in Assumption 1 which is assumed to hold throughout the paper.

ASSUMPTION 1: *Let the strategy space of player* $i = 1, 2$ *be given by* $R_i \in [0, \max\{v_1, v_2\}]$ *and* $h(R_i)$ *be differentiable and strictly concave over this interval with* $0 \leq h(R_i) < 1$, $h'(R_i) > 0$, *and* $h''(R_i) < 0$.

Examples of functional forms of $h(.)$ that satisfy Assumption 1 include the following:

$$h(R_i) = \frac{R_i + \psi}{R_i + 1}, 0 < \psi < 1, \tag{7}$$

$$h(R_i) = \sqrt{\frac{R_i}{K}}, 0 \leq R_i < K, \text{and } K > \max\{v_1, v_2\}. \tag{8}$$

Assumption 1 ensures that U^i is strictly concave with respect to R_i for any given R_j over the interval $R_i \in [0, \max\{v_1, v_2\}]$. Hence, when the first-order condition for maximization of U^i with respect to R_i holds, the second-order condition is always satisfied. In the subsequent sections we examine the equilibrium characteristics of contests involving both the symmetric and the asymmetric versions of the difference-form persuasion functions as in (1) and (2). While examining the asymmetric cases, we consider the effect each type of asymmetry can have on the equilibrium characteristics.

3. Equilibrium Behavior under Symmetric Difference-Form Persuasion Function

In this section, we examine contests involving the symmetric difference-form persuasion function given by (1). We assume $v_1 \geq v_2$ without loss of generality.

With simultaneous choice of resources, each player's decision problem involves maximizing her expected payoff U^i as given by (6) with respect to R_i taking R_j as given and $p_i(R_i, R_j)$ given by (1). Given Assumption 1, as long as $h'(0) > \frac{2}{\alpha v_i}$, $i = 1, 2$, the reaction functions of the two players are given by

$$h'(R_i^*) = \frac{2}{\alpha v_i} \text{ for } i = 1, 2. \tag{9}$$

The characteristics of the Nash equilibrium are presented in Proposition 1.[17]

PROPOSITION 1: *Under a symmetric difference-form persuasion function as in* (1) *when players choose their resources simultaneously:*

[17] See the Appendix for a proof.

(i) *The reaction function of each player is independent of the resources devoted by her rival and a dominant strategy equilibrium always exists.*

(ii) *If* $h'(0) > \frac{2}{\alpha v_i}$, $i = 1, 2$ *then the equilibrium involves both players investing in positive but potentially different level of resources with* $R_i^* = (h')^{-1}(\frac{2}{\alpha\, v_i})$ *. Along such an equilibrium,* R_i^* *increases with the level of evidence potency* α *and the player's own valuation of the prize* v_i*. It is invariant to changes in the rival player's valuation of its prize* v_j*.*

(iii) *If* $\frac{2}{\alpha v_2} \geq h'(0) > \frac{2}{\alpha v_1}$ *then the equilibrium involves* $R_1^* = (h')^{-1}(\frac{2}{\alpha\, v_1})$ *and* $R_2^* = 0$*.*

(iv) *If* $h'(0) \leq \frac{2}{\alpha v_i}$, $i = 1, 2$ *then in equilibrium neither player invests any resources toward the contest.*

(v) *When* $v_1 = v_2$*, the equilibrium is always symmetric with either both players investing the same positive level of resources into the contest, or both investing zero resources to it.*

(vi) *There is always partial rent dissipation in equilibrium as* $\sum_{i=1}^{2} R_i^* < \text{Max}\{v_1, v_2\}$*.*

Proposition 1 implies that unlike Baik (1998) and Che and Gale (2000), persuasion function (1) can support both corner and strictly interior pure strategy equilibria depending on the valuations of the prize and the sensitivity of $h(R_i)$ to resources.[18] Further, since each player is assured of a positive payoff even if she were to not expend any resources toward the contest regardless of the rival player's choice, it follows that rent dissipation is always partial. Interestingly, when each side has access to more compelling piece of evidence (higher α), it leads to a higher level of rent dissipation.

It is also interesting to note that the reaction function of each player is independent of the rival's effort (unlike Lazear and Rosen 1981; Baik 1998) due to the inherent nonstrategic nature of the contest. There are two implications from this. First, as per Proposition 1 (ii), the rival player's valuation of the prize has no impact on a player's equilibrium choice of resource spending. Therefore, preemption effect as defined by Che and Gale (2000) is absent.[19] Second, the equilibria of the simultaneous move game are identical to that of a sequential move game regardless of who moves first. These findings are summarized in Corollary 1.

COROLLARY 1: *Under a symmetric difference-form persuasion function as in* (1)*:*

(i) *There is no preemption effect as* $\sum_i R_i^*$ *is nondecreasing in* v_1*.*

(ii) *The equilibria of a simultaneous game are identical to those of a sequential game where one of the two players choose resource level first relative to the rival.*

With the symmetric persuasion function (1), the only source of asymmetry between the players is any difference between their stakes. To examine the implications of stake asymmetry on equilibrium behavior, we make the following assumption:

ASSUMPTION 2: *Let* $v_1 = v + \omega$, $v_2 = v - \omega$, *where* $v > 0$ *and* $|\omega| < v$*.*

In this case, the assumption of $v_1 \geq v_2$ is equivalent to assuming $\omega \geq 0$.

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[18] With linear resource costs, (1) is able to support strictly interior pure strategy equilibria, unlike Baik (1998) and Che and Gale (2000), because the strict concavity of $h(.)$ makes the expected payoffs of each player strictly concave in their resource spending. In principle, similar pure strategy equilibria could arise under Baik (1998) and Che and Gale (2000) if the resource cost function were assumed to be strictly convex.

[19] See footnote 11 for a comparison with Alcalde and Dahm (2007).

Using the representation in Assumption 2, we examine the implications of changes in evidence potency α and stake asymmetry ω on aggregate resource expenditures and aggregate welfare. The findings are presented in Proposition 2.

PROPOSITION 2: *Suppose that Assumption 2 and a symmetric difference-form persuasion function as in* (1) *apply. Then along the interior dominant strategy equilibrium of a simultaneous move game*

- *(i) An increase in α always increases aggregate resource spending. The impact of an increase in ω on aggregate resource spending is, however, ambiguous in general.*
- *(ii) Aggregate welfare always decreases with α if $\omega = 0$. When $\omega > 0$, then the impact of an increase in α and ω on aggregate welfare is ambiguous in general.*
- *(iii) When $h(.)$ is given by* (7)*, an increase in ω decreases aggregate resource spending and increases aggregate welfare. The effect of an increase in α on aggregate welfare is, however, ambiguous.*
- *(iv) When $h(.)$ is given by* (8)*, an increase in α increases aggregate welfare if ω is sufficiently high. An increase in ω increases aggregate resource spending and aggregate welfare.*

By inspecting (9) it is easy to appreciate that $\frac{\partial R_i^*}{\partial \alpha} > 0$ for $i = 1, 2$ given the strict concavity of $h(.)$ along the interior Nash equilibrium. From this it follows immediately that aggregate resource spending increases with α. To understand the implications for aggregate welfare $U = U^1(R_1, R_2) + U^2(R_1, R_2)$, note that from Equation (6) and Assumption 2 we have

$$U = v + \alpha \left[h(R_1) - h(R_2) \right] \omega - R_1 - R_2. \tag{10}$$

Hence, when $\omega = 0$, $U = v - R_1 - R_2$ so that as aggregate resources increase with α, U decreases with α. When $\omega > 0$, the impact of an increase in α on U is less clear cut. By differentiating (10) with respect to α and using the first-order conditions (9), we find that

$$\frac{dU^*}{d\alpha} = \left[h(R_1^*) - h(R_2^*) \right] \omega - \left(\frac{v-\omega}{v+\omega} \right) \frac{\partial R_1^*}{\partial \alpha} - \left(\frac{v+\omega}{v-\omega} \right) \frac{\partial R_2^*}{\partial \alpha}. \tag{11}$$

Note that since $\omega > 0$, it follows from (9) that $R_1^* > R_2^*$ and hence the first component in (11) is positive. This shows that by increasing the win probability of the player with the higher stake, an increase in α contributes positively to aggregate welfare. However, from Assumption 2 both $\frac{v-\omega}{v+\omega}$ and $\frac{v+\omega}{v-\omega}$ are positive, and $\frac{\partial R_i^*}{\partial \alpha} > 0$ for $i = 1, 2$. Hence, each of the remaining two components in (11) contributes to a decrease in aggregate welfare. This is because a higher α also stimulates higher aggregate spending. Hence, the overall welfare impact of an increase in α is in general ambiguous. However, as the case of $h(.)$ given by (8) suggests, it is plausible that if ω is sufficiently high, the first component might dominate causing aggregate welfare to increase with α.

When ω increases, it follows from Assumption 2, Equation (9), and strict concavity of $h(.)$ that $\frac{\partial R_1^*}{\partial \omega} > 0$ while $\frac{\partial R_2^*}{\partial \omega} < 0$. Hence, the net effect of an increase in ω on aggregate resource spending is in general ambiguous. To appreciate its impact on aggregate welfare, note from (10) that its direct contribution to aggregate welfare is positive as it tends to increase aggregate expected payoff since $R_1^* > R_2^*$ and so $h(R_1^*) > h(R_2^*)$. It also contributes positively to aggregate welfare by instigating Player 2 to cut her resource spending. However, since Player 1 is induced to increase her expenditure, this contributes negatively to aggregate welfare. As a result, the overall impact of an increase in ω on aggregate welfare is ambiguous. Interestingly, we get clear results for the case of an increase in ω for both the specific forms of $h(.)$ as given by (7) and (8). For $h(.)$ given by (7), an increase in ω decreases aggregate resource spending and increases welfare. For $h(.)$ given by (8), both aggregate spending and welfare increase with ω.[20] This possibility that an increase in asymmetry between players can lead to an increase in aggregate resource spending is surprising and in contrast to the Tullock contest case where, as shown in Konrad (2009), aggregate spending is inversely related to asymmetry between players.[21]

4. Equilibrium Characteristics under Asymmetric Persuasion Functions

As discussed briefly in Section 2, various factors can give rise to asymmetry in the persuasion function described in (5). In this section, we will explore the implication of each of those sources of asymmetry for equilibrium behavior of players. We begin with the case of asymmetry in the evidence production process.

4.1. Asymmetric Evidence Production and Its Impact on Equilibrium Spending

To focus on the effect of asymmetric evidence production, in this subsection the following assumption will apply:

ASSUMPTION 3: *Let* $\gamma = \frac{1}{2}$, $0 < (\frac{\Gamma-1}{\Gamma}) = (\frac{1-\delta}{\delta}) = \alpha < 1$, *and* $v_1 = v_2 = v$. *However, let* $h_1(R_1) = \theta\, h(R_1)$ *and* $h_2(R_2) = (1-\theta)\, h(R_2)$, *where* $h(.)$ *follows Assumption 1, and* $\theta \in (0, 1)$ *with* $\theta \neq \frac{1}{2}$.

Assumption 3 ensures that the asymmetry in the persuasion contest is purely due to differences in the evidence production probabilities so that $h_1(R_1) \neq h_2(R_2)$ when $R_1 = R_2$. This is due to $\theta \neq \frac{1}{2}$. Following Hirshleifer and Osborne (2001) and Robson and Skaperdas (2008), θ can be interpreted as the *degree of truth* or, in the case of litigation, as the *level of property rights protection.* For example, if the truth (or property rights) is with Player 1, then $\theta \in (\frac{1}{2}, 1)$, so that when $R_1 = R_2 = R, h_1(R) > h_2(R)$. This implies that the side arguing for the truth (Player 1 in this instance) will have a higher chance of getting favorable evidence when both players invest the same level of resources. The closer θ is to 1 (or the better defined property rights are), the easier it is to argue for Player 1, who is on the side of the truth. Analogously, if the truth were with Player 2, then $\theta \in (0, \frac{1}{2})$. For the sake of brevity, we will assume that $\theta \in (\frac{1}{2}, 1)$.

Given Assumption 3, the persuasion function in Equation (5) reduces to

$$p_1(R_1, R_2) = \frac{1}{2} + \frac{\alpha}{2}\left[\theta\, h(R_1) - (1-\theta)\, h(R_2)\right]. \tag{12}$$

Each player *i*'s decision problem still involves choosing an appropriate level of resource spending to maximize her expected payoff U^i given by (6) except that $p_i(R_1, R_2)$

[20] The reasoning for the different impact of an increase in ω on aggregate resource spending for the two function forms of $h(.)$ stems from the observation that equilibrium resource expenditures are concave with respect to v_i for (7) while the opposite is true for (8) where they are convex with respect to v_i.

[21] See Konrad (2009, p. 46). In the Tullock contest, greater symmetry contributes to higher aggregate spending by increasing the incentive to invest for both players.

is given by (12). Using the first-order conditions, the reaction functions of the two players are

$$R_1^* = \begin{cases} (h')^{-1}\left(\frac{2}{\alpha\theta v}\right) \text{ when } h'(0) > \frac{2}{\alpha\theta v} \\ 0 \text{ otherwise} \end{cases}, \tag{13}$$

$$R_2^* = \begin{cases} (h')^{-1}\left(\frac{2}{\alpha(1-\theta)v}\right) \text{ when } h'(0) > \frac{2}{\alpha(1-\theta)v} \\ 0 \text{ otherwise} \end{cases}. \tag{14}$$

By inspection of (13) and (14), it is clear that the reaction functions of both players are analogous to the case of asymmetric prize valuations where $v_1 > v_2$, as examined in Section 3. Hence, Proposition 1 ((i)–(iv) and (vi)) continues to apply qualitatively and so does the invariance of Nash equilibrium to sequential moves as stated in Corollary 1. Proposition 1 (v) no longer holds as the equilibrium spending differs between players as discussed in Proposition 3.

PROPOSITION 3: *When Assumption 3 applies and* $\theta \in (\frac{1}{2}, 1)$, *the truth is on the side of Player 1 who puts in more resources in equilibrium and has a higher probability of winning along a strict interior equilibrium. The closer* θ *is to 1, the greater is this effect. When the equilibrium consists of only one of the two players actively spending resources in the contest, Player 1 is the active player.*[22]

Proposition 3 follows from the observation that when $\theta \in (\frac{1}{2}, 1)$, we have $\frac{2}{\alpha\theta v} < \frac{2}{\alpha(1-\theta)v}$. It indicates that when the only source of asymmetry is a tilt in the evidence production toward the player arguing for the truth, this natural advantage to that player gets reinforced through the equilibrium choice of resources. Note that this is different from the Nash–Cournot equilibrium behavior in Hirshleifer and Osborne (2001), who examine a ratio-form asymmetric contest and find that both parties always put in equal resources in equilibrium regardless of the level of the degree of truth. It is also apparent that the general properties outlined in Proposition 2 regarding implications of changes in α and ω continue to hold qualitatively.

4.2. Bias in the Decision Threshold and Its Impact on Equilibrium Behavior

To study the effect of a bias in the decision threshold, in this subsection we allow for $\gamma \neq \frac{1}{2}$ while suppressing other sources of asymmetry as stated in Assumption 4.

ASSUMPTION 4: *Let* $\frac{1}{1+\alpha} \geq \gamma \neq \frac{1}{2}$, $0 < (\frac{\Gamma-1}{\Gamma}) = (\frac{1-\delta}{\delta}) = \alpha < 1$, $v_1 = v_2 = v$ *and* $h_1(.) = h_2(.) = h(.)$.[23]

With Assumption 4, the persuasion function in (5) reduces to

$$p_1(R_1, R_2) = (1-\gamma) + \gamma\alpha\left[h(R_1) - h(R_2)\right]. \tag{15}$$

Since $0 \leq h(.) < 1$, as long as $\gamma \leq \frac{1}{1+\alpha}$, (15) is naturally bounded between 0 and 1 for any (R_1, R_2).

[22] It is worthwhile to note, however, that when the valuations of the prize are asymmetric, this effect can be potentially dominated if the player who values the prize higher is not the one arguing for the truth.
[23] Since $0 < \alpha < 1$, $\frac{1}{2} < \frac{1}{1+\alpha} < 1$.

Note that when $\gamma > \frac{1}{2}$ the decision threshold favors Player 2 as the bar is higher for Player 1 to prove her case relative to Player 2. From (15), this implies that $p_1(R_1, R_2) < p_2(R_1, R_2)$ when $R_1 = R_2$. The opposite is true when $\gamma < \frac{1}{2}$. To illustrate the implications of such threshold bias, we use (15) to construct each player's expected payoff in the game as given by (6). Given players' objectives, the maximization of the expected payoffs by the players via simultaneous choice of resources leads to the following reaction functions:

$$R_i^* = \begin{cases} (h')^{-1}\left(\frac{1}{\alpha\gamma v}\right) \text{when } h'(0) > \frac{1}{\alpha\gamma v} \\ 0 \text{ otherwise} \end{cases} \quad \text{for } i = 1, 2. \tag{16}$$

Note that the asymmetry in the persuasion function due to $\gamma \neq \frac{1}{2}$ does not lead to an asymmetry in equilibrium expenditures. This leads us to Proposition 4.

PROPOSITION 4: *When Assumption 4 applies, asymmetry in the persuasion function due to threshold bias does not lead to asymmetry in equilibrium spending as both players invest the same amount of resources in the contest. However, the threshold bias does affect the level of equilibrium spending and therefore rent dissipation both of which increase with γ along the strictly interior equilibrium.*

Proposition 4 follows from condition (16) and the strict concavity of $h(.)$ which imply that $\frac{\partial R_i^*}{\partial \gamma} > 0$, $i = 1, 2$. A comparison of Proposition 4 with Proposition 3 reveals that asymmetries due to degree of truth and threshold bias have distinct effects on the equilibrium behavior of the two contestants. Proposition 1 continues to apply qualitatively except that the reaction functions are given by (16), and similarly the invariance of Nash equilibrium to sequential choice of resources by the players as stated in Corollary 1 continues to hold. The general properties outlined in Proposition 2 also continue to hold qualitatively.

4.3. Evidence Bias and Its Impact on Equilibrium Behavior

We now allow for differences in the potency of evidence presented by the two contestants so that $(\frac{\Gamma-1}{\Gamma}) \neq (\frac{1-\delta}{\delta})$. This may be due to one player naturally having access to a more convincing piece of evidence than the other. It could also be a form of bias where the audience is more receptive toward the evidence presented by one of the two contestants. In this section, we examine the impact of such evidence bias on players' equilibrium behavior. As with previous cases, we suppress other sources of asymmetry as is stated in Assumption 5.

ASSUMPTION 5: *Let* $\alpha_1 = (\frac{\Gamma-1}{\Gamma}) \neq (\frac{1-\delta}{\delta}) = \alpha_2$, $\gamma = \frac{1}{2}$, $v_1 = v_2 = v$, *and* $h_1(.) = h_2(.) = h(.)$. *Further* $0 < \alpha_2 < \alpha_1 < 1$ *so that the evidence bias is in favor of Player 1.*[24]

When Assumption 5 applies, the persuasion function in (5) becomes

$$p_1(R_1, R_2) = \frac{1}{2} + \frac{1}{2}\left[\alpha_1 h(R_1) - \alpha_2 h(R_2) - (\alpha_1 - \alpha_2) h(R_1) h(R_2)\right]. \tag{17}$$

[24] The case of $\alpha_1 < \alpha_2$ is analytically identical and therefore not discussed for the sake of brevity.

The win probability of player 2 is $p_2(R_1, R_2) = 1 - p_1(R_1, R_2)$. Note that the evidence bias $(\alpha_1 - \alpha_2)$ works its way via a cross-product term $h(R_1)h(R_2)$ with a negative coefficient for Player 1 which makes (17) distinct from other asymmetries examined so far. Lemma 1 identifies the basic characteristics of the persuasion function in (17):

LEMMA 1:

(i) $\frac{\partial p_i}{\partial \alpha_i} = \frac{h(R_i)(1-h(R_j))}{2} > 0$ *for* $i, j = 1, 2, i \neq j$.

(ii) $\frac{\partial p_i}{\partial \alpha_j} = -\frac{h(R_j)(1-h(R_i))}{2} < 0$ *for* $i, j = 1, 2, i \neq j$.

(iii) $\frac{\partial p_1}{\partial R_1} = \frac{1}{2}[\alpha_1 - (\alpha_1 - \alpha_2)h(R_2)]h'(R_1) > 0$ *and* $\frac{\partial p_2}{\partial R_2} = \frac{1}{2}[\alpha_2 + (\alpha_1 - \alpha_2)h(R_1)]h'(R_2) > 0$.

(iv) $\frac{\partial p_1}{\partial R_2} = -\frac{1}{2}[\alpha_2 + (\alpha_1 - \alpha_2)h(R_1)]h'(R_2) < 0$ *and* $\frac{\partial p_2}{\partial R_1} = -\frac{1}{2}[\alpha_1 - (\alpha_1 - \alpha_2)h(R_2)]h'(R_1) < 0$.

(v) $\frac{\partial^2 p_1}{\partial R_1^2} = \frac{1}{2}[\alpha_1 - (\alpha_1 - \alpha_2)h(R_2)]h''(R_1) < 0$ *and* $\frac{\partial^2 p_2}{\partial R_2^2} = \frac{1}{2}[\alpha_2 + (\alpha_1 - \alpha_2)h(R_1)]h''(R_2) < 0$.

(vi) $\frac{\partial^2 p_1}{\partial R_1 \partial R_2} = -\frac{1}{2}[(\alpha_1 - \alpha_2)h'(R_2)]h'(R_1) < 0$ *and* $\frac{\partial^2 p_2}{\partial R_1 \partial R_2} = \frac{1}{2}[(\alpha_1 - \alpha_2)h'(R_1)]h'(R_2) > 0$.

(vii) $p_1(R_1, R_2) > p_2(R_1, R_2)$ *when* $R_1 = R_2$ *and* $\alpha_1 - \alpha_2 > 0$.

Properties (i) and (ii) imply that each player's win probability is increasing in the strength of its evidence and decreasing in that of its rival. Property (vii) implies that when both players put in the same level of resources the evidence bias (in favor of Player 1) provides an advantage to Player 1 in terms of a higher win probability. These properties suggest that, despite the negative coefficient in the cross-product term for Player 1, the first-order effects of the evidence bias are to enhance the win probability of Player 1. However, the evidence bias does have the opposite effect on the marginal incentive to put in resources toward the contest for Player 1. This can be first appreciated by observing the marginal impact of resources on the win probabilities in (iii). Note that the evidence bias is reducing the marginal return to Player 1's investment in the contest while it is adding to that of Player 2. Further, note that as per (vi), when $\alpha_1 - \alpha_2 > 0$, an increase in the rival's investment of resources toward the contest discourages Player 1 while it encourages Player 2 to increase its resources. These two properties imply that the evidence bias strengthens the weaker player's marginal incentive to put in resources relative to the stronger player. Intuitively, the weaker player is tempted to invest more at the margin in an attempt to compensate for the evidence bias against her. As we will observe subsequently, this property manifests itself also via differences in the shapes of the reaction functions for the two players. The sign of the partial derivatives described in properties (iii)–(vi) follow straightforwardly from our assumption of monotonicity and strict concavity of $h(.)$.

To see how the above characteristics of the persuasion function (17) impact on equilibrium behavior, recall that in the Cournot game each player i aims to maximize U^i (determined by (6) and (17)) through her choice of $R_i \in [0, v]$, taking R_j as given where $i, j = 1, 2, i \neq j$. Since $U^i > 0$ at $R_i = 0$ and $U^i < 0$ at $R_i = v$ it follows that $R_i^* < v$. The sufficient condition for a strictly interior Nash equilibrium in this game is provided by Lemma 2.

LEMMA 2: *Given Assumption 5, if* $h'(0) > \frac{2}{\alpha_2 v}$, *then* $0 < R_i^* < v, i = 1, 2$ *so that both players will always invest positive level of resources into the contest regardless of the level invested by the rival.*

From Lemma 2 it follows that if $h'(0) > \frac{2}{\alpha_2 v}$, players' best responses are strictly interior and given by the first-order conditions as follows:

$$h'(R_1) = \frac{2}{[\alpha_1 - (\alpha_1 - \alpha_2)h(R_2)]\, v}, \tag{18}$$

$$h'(R_2) = \frac{2}{[\alpha_2 + (\alpha_1 - \alpha_2)h(R_1)]\, v}. \tag{19}$$

Equation (18) represents the reaction function for Player 1 while (19) represents that of Player 2. Since U^i is strictly concave in R_i over the strategy space $[0, v]$ $i = 1, 2$ due to Assumption 1, the second-order conditions are always satisfied along (18) and (19). Note that the evidence bias enters negatively in the reaction function of Player 1. This coupled with strict concavity of $h(.)$ implies that as R_2 increases, the optimal level of R_1 falls. Hence, the reaction function of Player 1 is negatively sloped. However, for Player 2, the evidence bias enters positively in the denominator which implies that as R_1 increases, the optimal level of R_2 increases giving a positive slope to the reaction function of Player 2. These reactions functions suggest that the evidence bias makes Player 2 aggressive at the margin relative to Player 1. Since each player's expected payoff is strictly positive even if she does not put any resources into the contest, it follows that in any Nash equilibrium, it will always be the case that $\nu > R_1^* + R_2^*$ so that the rent dissipation will be partial. The equilibrium behavior is summarized in Proposition 5.[25]

PROPOSITION 5: *Under Assumption 5 and* $h'(0) > \frac{2}{\alpha_2 v}$, *the Cournot equilibrium behavior is as follows:*

- *(i)* *Player 1's optimal resource expenditure is strictly positive as given by her reaction function* (18) *and is inversely related to the resource expenditure of Player 2.*
- *(ii)* *Player 2's optimal resource expenditure is strictly positive as given by her reaction function* (19) *and is positively related to the resource expenditure of Player 1.*
- *(iii)* *A unique interior pure strategy Cournot–Nash equilibrium exists and determined by the crossing point of the reaction functions of the two players.*
- *(iv)* *In principle, the unique pure-strategy Cournot–Nash equilibrium can be one of three kinds:* (1) *symmetric equilibrium with* $R_1^* = R_2^*$, (2) $R_1^* > R_2^*$ *so that the player favored by the evidence bias puts a higher effort in equilibrium, and* (3) $R_1^* < R_2^*$ *so that the weaker player puts in greater effort to counterbalance the evidence bias in equilibrium.*
- *(v)* *There is partial rent dissipation in any Cournot–Nash equilibrium.*
- *(vi)* *The equilibrium outlays differ in a Stackelberg game relative to the Cournot game. When Player 1 moves first, she is induced to reduce her expenditure relative to that in the Cournot game. When Player 2 moves first, she is induced to increase her expenditure relative to that in the Cournot game.*

[25] Proofs of Proposition 5 parts (i) and (ii) follow immediately from inspection of (18) and (19) via the preceding discussion. For proofs of parts (iii)–(vi), please see the Appendix.

The conditions under which the symmetric Cournot equilibrium $R_1^* = R_2^* = R^*$ holds are given by

$$h(R^*) = \frac{1}{2}, \tag{20}$$

$$h'(R^*) = \frac{4}{[(\alpha_1 + \alpha_2)]\, v}. \tag{21}$$

It is apparent that the level of R^* implied by (21) depends on the values of α_1, α_2, and v and is very unlikely to be consistent with (20). Hence, in general, the symmetric equilibrium is unlikely and easily disturbed by small changes in either α_1, α_2, or v. Therefore in most instances, the Cournot equilibrium will be asymmetric with either $R_1^* > R_2^*$ or $R_1^* < R_2^*$.

When the Cournot equilibrium is such that $R_1^* > R_2^*$, it immediately follows that $p_1(R_1^*, R_2^*) > p_2(R_1^*, R_2^*)$ (given Lemma 1 (vii)). Hence, along such equilibrium, the player favored by the evidence bias also puts in more resources into the contest and therefore has a higher equilibrium probability of winning. In this instance, the equilibrium choices of resource expenditures by both players reinforce the advantage conferred to Player 1 through the evidence bias. The necessary conditions for such equilibrium are

$$1 > h(R_1^*) + h(R_2^*), \tag{22}$$

$$h(R_1^*) > h(R_2^*). \tag{23}$$

When $R_1^* < R_2^*$, the weaker player puts in greater effort in the Cournot equilibrium and at least partially offsets the disadvantage of the evidence bias favoring the rival. In this case, it is theoretically possible that $p_1(R_1^*, R_2^*) < p_2(R_1^*, R_2^*)$. If this were to happen, it represents a case where the advantage conferred to Player 1 through evidence bias is overwhelmed by the greater marginal incentive to put in resources on Player 2's part. It is useful to recall that while the evidence bias lowers the win probability of the weaker player for given levels of resources, it also provides a higher marginal incentive to her to compete in the contest. When $p_1(R_1^*, R_2^*) < p_2(R_1^*, R_2^*)$, the latter effect dominates. The necessary conditions for a Cournot equilibrium with $R_1^* < R_2^*$ are

$$1 < h(R_1^*) + h(R_2^*), \tag{24}$$

$$h(R_1^*) < h(R_2^*). \tag{25}$$

By examining conditions (22)–(25), it is apparent that characteristics of the $h(.)$ function will determine which one of the two types of asymmetric Cournot equilibria will eventuate. This point is illustrated by Figures 1 and 2 which plot the reaction functions of the two players for $h(.)$ given (7) and (8) using specific parameter values that satisfy Lemma 2 and Assumption 5.

Figure 1 is drawn for a specific case of $h(.)$ given by (8) and illustrates the possibility of Cournot equilibrium involving $R_1^* > R_2^*$. On the other hand, Figure 2 is drawn for a specific case of $h(.)$ given by (7) and illustrates the possibility of Cournot equilibrium involving $R_1^* < R_2^*$. Corollary 2 identifies sufficient conditions under which $R_1^* < R_2^*$ for $h(.)$ given by (7).[26]

[26] Similarly, note that under the conditions in Proposition 6, the Cournot equilibrium always involves $R_1^* > R_2^*$ for $h(.)$ given by (8).

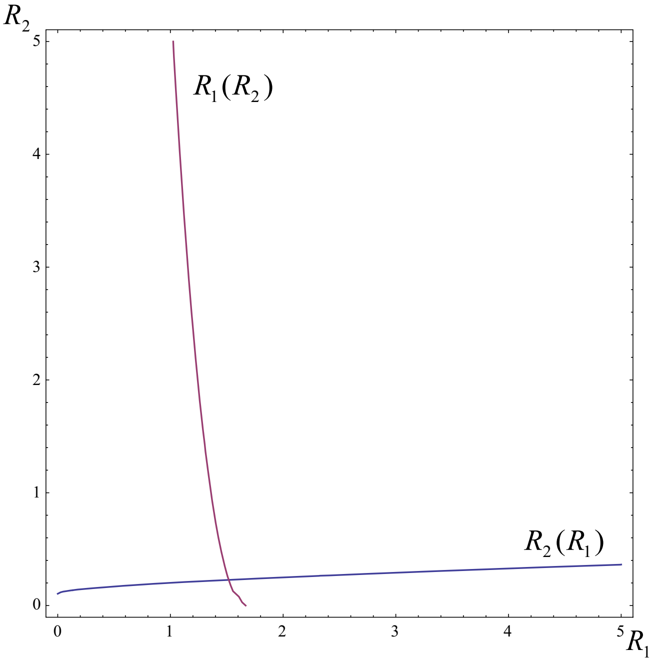

Figure 1: Players' reaction functions under Assumption 5 when $h(.)$ is given by (8), $\alpha_1 = 0.8, \alpha_2 = 0.2$, $K = 60$, $v = 50$.

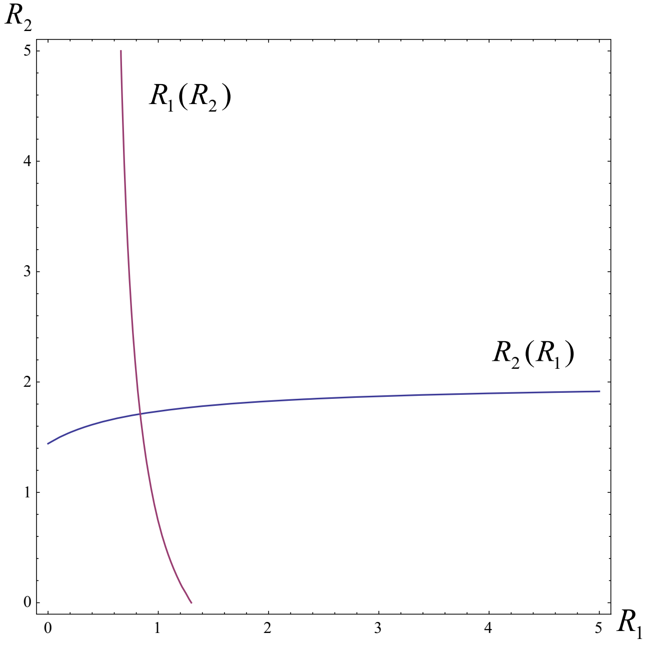

Figure 2: Players' reaction functions under Assumption 5 when $h(.)$ is given by (7), $\alpha_1 = 0.8, \alpha_2 = 0.2$, $\psi = 0.55$, $v = 50$.

COROLLARY 2: *Under Assumption 5, and* $h(.)$ *given by* (7), *any interior Cournot–Nash equilibrium will involve* $R_1^* < R_2^*$ *if* $\frac{1}{2} < \psi < 1 - \frac{2}{\alpha_2 v}$ *and* $\alpha_2 v > 4$.

Indeed, when (7) holds and $\psi > \frac{1}{2}$, neither (20) nor (22) can be satisfied and therefore the interior Cournot equilibrium can only involve $R_1^* < R_2^*$ in which the disadvantaged player is associated with higher resource expenditure. The sufficient conditions for such an interior Cournot equilibrium to exist (so that the condition in Lemma 2 is satisfied) require that v is adequately large while ψ is not too high. These two restrictions ensure that both players have enough marginal incentive to invest positively toward the contest.

It is also worth noting that in contrast to the equilibria generated by the symmetric persuasion function (1), the equilibrium resource spending of either player is sensitive to the order in which players choose their expenditures when the persuasion function is given by (17) (and stated in Proposition 5 (vi)). This is due to players' reaction functions no longer being flat as in the symmetric case. When Player 1 chooses her expenditure first, she is induced to cut her expenditure relative to the Cournot equilibrium. While a marginal cut in her expenditure has no direct impact on her payoff, it induces Player 2 to reduce her expenditure as her reaction function is positively sloped (as stated in Proposition 5 (ii) and illustrated in Figures 1 and 2). Player 1 benefits from this reaction through a marginal increase in her win probability. When Player 2 chooses her expenditure first, exactly conversely she is induced to increase her expenditure relative to the Cournot equilibrium. This is because she benefits from an increase in her win probability from Player 1's response of cutting back her expenditure as her reaction function is downward sloping (as stated in Proposition 5 (i) and illustrated in Figures 1 and 2). Hence, when Player 1 is the "favorite" at the Cournot equilibrium ($p_1^* > p_2^*$ as is the case when $R_1^* \geq R_2^*$), she chooses to cut back on her expenditure when acting as a Stackelberg leader. This is in contrast to the favorite's tendency to increase her resource spending when given an opportunity to precommit under the asymmetric logit and probit contest functions examined in Dixit (1987). Similarly, the behavior of the "underdog" Player 2 when acting as a Stackelberg leader is opposite to that in Dixit (1987). These differences arise as reaction functions of the favorite and underdog in Dixit (1987) have opposite gradients locally around the Cournot equilibrium to those generated by (17).

We now turn to some comparative statics involving (17) and invoke Assumption 6 which presents a convenient way to represent the restrictions imposed by Assumption 5 on α_1 and α_2.

ASSUMPTION 6: *Let* $\alpha_1 = \alpha + \Delta$, $\alpha_2 = \alpha - \Delta$ *where* $0 < \Delta < \alpha < 1$.

Using Assumptions 2 and 6, we study the impact of changes in evidence potency α, evidence asymmetry Δ and stake asymmetry ω on equilibrium expenditures and aggregate welfare. Similar to the findings presented in Proposition 2 for the persuasion function (1), we find that for the general case of $h(.)$ governed by Assumption 1 the impact of an increase in α, Δ and ω on aggregate resource spending and welfare is ambiguous for (17).[27] However, for $h(.)$ given by (7) and (8), under certain conditions some clear results emerge which are presented in Propositions 6 and 7 below.

[27] These findings are qualitatively very similar to those presented in Proposition 2. Hence, for the sake of brevity, they are discussed only in the online Supporting Information Appendix.

PROPOSITION 6: *Under Assumptions 2 and 6, persuasion function* (17) *and* $h(.)$ *given by* (7), *when* $\omega = 0$ *and the conditions in Corollary 2 hold:*

(i) An increase in α *leads to an increase in aggregate resource spending and a decrease in aggregate welfare.*

(ii) An increase in Δ *leads to a decrease in aggregate resource spending and an increase in aggregate welfare.*

PROPOSITION 7: *Under Assumptions 2 and 6, persuasion function* (17) *and* $h(.)$ *given by* (8)*:*

(i) Cournot–Nash equilibrium is always interior with $R_1^* > R_2^*$.

(ii) If $\omega = 0$, *an increase in* α *leads to an increase in aggregate resource spending and a decrease in aggregate welfare.*

(iii) If $\omega = 0$, *an increase in* Δ *leads to an increase in aggregate resource spending and a decrease in aggregate welfare.*

(iv) If the stake asymmetry is high so that ω *is arbitrarily close to* v, *an increase in* ω *leads to an increase in aggregate resource spending and aggregate welfare.*

Both Propositions 6 and 7 suggest that the effects of an increase in α on aggregate resource spending and aggregate welfare when (17) applies are qualitatively the same as those for the symmetric persuasion function (Proposition 2), at least when $h(.)$ is given by (7) or (8), and there is no prize asymmetry. An increase in evidence asymmetry Δ seems to reduce aggregate spending and therefore increase aggregate welfare when $h(.)$ is given by (7) but the opposite is true when $h(.)$ is given by (8). Proposition 7 also implies that, when persuasion function is given by (17) and $h(.)$ is given by (8), Player 1, who has stronger evidence, always spends more resources relative to her rival in the Cournot equilibrium and is therefore always the favorite. Further, as with Proposition 2 (iv), an increase in asymmetry between players (either through an increase in Δ or through an increase in ω) can *increase* aggregate resource spending under certain conditions.[28]

5. Conclusion

We have examined two-player difference-form contests that are best thought of as "persuasion functions"; that is, as applying to instances, such as litigation, lobbying, or political campaigning, in which different parties expend resources in order to persuade an audience. Contrary to specific cases of difference-form contests examined by Baik (1998) and Che and Gale (2000), we have found that these contests can support both corner and pure-strategy interior equilibria and at least in the form in (1) they are simple to derive. The equilibrium behavior of contestants may not satisfy the "preemption" property defined in Che and Gale (2000), suggesting that it is not a general property of difference-form contests. Further, in the case of asymmetric forms some counterintuitive outcomes are possible. The player with the weaker evidence may have a stronger marginal incentive to spend resources so that the precommitment behavior of the favorite and underdog in sequential move contests involving (17) can be opposite to Dixit

[28] For the specific case of $h(.)$ given by (7), the impact of an increase in stake asymmetry on aggregate resource spending and aggregate welfare is ambiguous.

(1987). In particular, the favorite may choose to precommit to lower expenditures relative to her chosen level in a Cournot–Nash equilibrium.

Further, since the functional forms examined in the paper are explicitly characterized as outcomes of a persuasion contest, the parameters of the functions carry natural inferential interpretations such as evidence potency, bias, and truth. This allows us to uncover their impact on equilibrium spending in a simple framework. We find that increase in evidence potency may intensify resource expenditures especially in the symmetric case. A bias in the decision threshold does not lead to asymmetry in equilibrium spending but alters the intensity of the spending. An *a priori* bias in favor of truth translates into the truthful side spending more resources than the rival which reinforces the initial advantage. An evidence bias, however, may have the opposite effect where the weaker party puts in more resources in equilibrium to counter the evidence-based disadvantage. Further, unlike the Tullock contest, in some circumstances increased asymmetry between players (such as differences in stakes) may lead to increased aggregate resource expenditures. Given the relative paucity of applied studies which have used a difference-form contest function, with the exception of Besley and Persson (2009) these findings suggest that it would be worthwhile to reexamine many applied settings that involve persuasion using these functions and determine whether they shed a different light than those that come from existing studies.

A limitation of the functional forms examined in the paper is that they represent only two -player contests, unlike those examined in Alcalde and Dahm (2007) and Bevia and Corchon (2015). However, this still leaves room for various applications, given that many real-world persuasion contests often involve two competing parties, as in litigation and electoral contests. Further, in principle, the additive form in (1) can be easily extended to $N > 2$ players. However, it remains to be seen whether the parameters of the extended function can be interpreted in an intuitive way through a persuasion-based microfoundation.

Appendix[29]

Proof of Proposition 1: Let us examine the payoff function of player i in Section 3 which is reproduced below:

$$U^i(R_i, R_j) = \left\{ \frac{1}{2} + \frac{\alpha}{2}\left[h(R_i) - h(R_j)\right] \right\} v_i - R_i \text{ for } i,\, j = 1, 2 \text{ and } i \neq j. \quad \text{(A1)}$$

Equation (A1) can be rearranged as

$$U^i = \frac{1}{2}\, v_i\{1 - \alpha h(R_j)\} + \frac{\alpha}{2}\, v_i\, h(R_i) - R_i. \quad \text{(A2)}$$

Note that the above payoff is additively separable in R_i and R_j. From this it follows that the optimal choice of R_i by player i will be independent of R_j and will involve a dominant strategy. This proves part (i).

Recall that $U^i > 0$ at $R_i = 0$ and $U^i < 0$ at $R_i = v_i$. Assumption 1 ensures that U^i is strictly concave over the strategy space $R_i \in [0, \max\{v_1, v_2\}]$. Hence, if $h'(0) > \frac{2}{\alpha v_i}$ then

[29] Throughout this section, where necessary, we abbreviate $h(R_i) = h^i$, $h'(R_i) = h'^i$, and $h''(R_i) = h''^i$ for $i = 1, 2$ for ease of exposition.

$R_i^* > 0$ and it is given by the first-order condition $h'(R_i^*) = \frac{2}{\alpha v_i}$ so that $R_i^* = (h')^{-1}(\frac{2}{\alpha v_i})$. From this it also follows that $\frac{\partial R_i^*}{\partial \alpha}$ and $\frac{\partial R_i^*}{\partial v_i}$ are positive and $\frac{\partial R_i^*}{\partial v_j} = 0$ for $i, j = 1, 2, i \neq j$. When $h'(0) \leq \frac{2}{\alpha v_i}$, then $R_i^* = 0$. Parts (ii)–(v) follow straightforwardly from this result.

Since $U^i > 0$ when $R_i = 0$ regardless of the level of R_j it must be the case that at $R_i{}^* > 0$, $U^{*i} > 0$. Hence, in any pure strategy equilibrium, it must be the case that

$$U^{*1} = \left\{\frac{1}{2} + \frac{\alpha}{2}\left[h(R_1^*) - h(R_2^*)\right]\right\} v_1 - R_1^* > 0,$$

$$U^{*2} = \left\{\frac{1}{2} + \frac{\alpha}{2}\left[h(R_2^*) - h(R_1^*)\right]\right\} v_2 - R_2^* > 0.$$

Since by assumption, $v_1 \geq v_2$, it follows from the above two conditions that $R_1^* + R_2^* < v_1$ implying partial dissipation of rents. This proves part (vi). ■

Proof of Proposition 2: From Proposition 1 (ii), we know that $\frac{\partial R_i^*}{\partial \alpha} > 0$ for $i = 1, 2$. Hence, it follows immediately that $\frac{\partial (R_1^* + R_2^*)}{\partial \alpha} > 0$. Recall that the first-order conditions are

$$h'^1 = \frac{2}{\alpha(v + \omega)}, \tag{A3}$$

$$h'^2 = \frac{2}{\alpha(v - \omega)}. \tag{A4}$$

From strict concavity of $h(.)$, it follows by inspecting (A3) and (A4) that while $\frac{\partial R_1^*}{\partial \omega} > 0$, $\frac{\partial R_2^*}{\partial \omega} < 0$. Hence, we cannot sign $\frac{\partial (R_1^* + R_2^*)}{\partial \omega}$ unambiguously. This proves part (i).

When $\omega = 0$, from (10) we get $U = v - R_1 - R_2$. Since $\frac{\partial (R_1^* + R_2^*)}{\partial \alpha} > 0$, it follows that $\frac{\partial U^*}{\partial \alpha} < 0$.

When $\omega > 0$, by differentiating (10) at (R_1^*, R_2^*) with respect to α it follows that

$$\frac{dU^*}{d\alpha} = [h(R_1^*) - h(R_2^*)]\omega + (\alpha\omega h'^1 - 1)\frac{dR_1^*}{d\alpha} - (\alpha\omega h'^2 + 1)\frac{dR_2^*}{d\alpha}. \tag{A5}$$

Using the first-order conditions as given by (A3) and (A4), (A5) simplifies to

$$\frac{dU^*}{d\alpha} = [h(R_1^*) - h(R_2^*)]\omega - \left(\frac{v - \omega}{v + \omega}\right)\frac{\partial R_1^*}{\partial \alpha} - \left(\frac{v + \omega}{v - \omega}\right)\frac{\partial R_2^*}{d\alpha}. \tag{A6}$$

Observe that $(\frac{v-\omega}{v+\omega}) > 0$, $(\frac{v+\omega}{v-\omega}) > 0$ while $\frac{\partial R_i^*}{\partial \alpha} > 0$ for $i = 1, 2$. Further from strict concavity of $h(.)$ and the first-order conditions (A3) and (A4) it follows that $R_1^* > R_2^*$. Given this it is apparent that while the first component of (A6) is positive, the other two components are negative. Hence, when $\omega > 0$, the sign of $\frac{dU^*}{d\alpha}$ is generally ambiguous.

By differentiating (10) at (R_1^*, R_2^*) with respect to ω it follows that

$$\frac{dU^*}{d\omega} = \alpha[h(R_1^*) - h(R_2^*)] + (\alpha\omega h'^1 - 1)\frac{\partial R_1^*}{\partial \omega} - (\alpha\omega h'^2 + 1)\frac{\partial R_2^*}{\partial \omega}. \tag{A7}$$

Using the first-order conditions as given by (A3) and (A4), (A7) simplifies to

$$\frac{dU^*}{d\omega} = \alpha[h(R_1^*) - h(R_2^*)] - \left(\frac{v - \omega}{v + \omega}\right)\frac{\partial R_1^*}{\partial \omega} - \left(\frac{v + \omega}{v - \omega}\right)\frac{\partial R_2^*}{\partial \omega}. \tag{A8}$$

Since $R_1^* > R_2^*$ (and therefore from monotonicity of $h(.)$, $h(R_1^*) > h(R_2^*)$), $\frac{v-\omega}{v+\omega}$ and $\frac{v+\omega}{v-\omega}$ are positive, and $\frac{\partial R_1^*}{\partial \omega} > 0$ while $\frac{\partial R_2^*}{\partial \omega} < 0$ it follows that the first and the last components in (A8) are positive while the second component is negative. Hence, the overall sign of $\frac{dU^*}{d\omega}$ cannot be determined unambiguously.

This proves part (ii).

When $h(R_i)$ is given by (7), maximization of U^i as given by (6) with respect to R_i $i = 1, 2$ yields

$$R_1^* = \sqrt{\frac{\alpha(1-\psi)(v+\omega)}{2}} - 1, \tag{A9}$$

$$R_2^* = \sqrt{\frac{\alpha(1-\psi)(v-\omega)}{2}} - 1. \tag{A10}$$

By inspection of (A9) and (A10), it is apparent that for sufficiently high v an interior equilibrium exists with $R_i^* > 0$ for $i = 1, 2$. Using (A9) and (A10) and differentiating with respect to ω we get

$$\frac{\partial(R_1^* + R_2^*)}{\partial \omega} = \frac{1}{2}\sqrt{\frac{\alpha(1-\psi)}{2}}\left(\frac{1}{\sqrt{v+\omega}} - \frac{1}{\sqrt{v-\omega}}\right) < 0.$$

Recall that the first component on the R.H.S. of (A8) is always positive when $\omega > 0$.

Now, since $\frac{\partial R_1^*}{\partial \omega} > 0$, $\frac{\partial R_2^*}{\partial \omega} < 0$, and $\frac{\partial(R_1^*+R_2^*)}{\partial \omega} < 0$, it follows that $(\frac{v-\omega}{v+\omega})\frac{\partial R_1^*}{\partial \omega} + (\frac{v+\omega}{v-\omega})\frac{\partial R_2^*}{\partial \omega} < 0$. Hence, R.H.S. of (A8) must be strictly positive so that $\frac{dU^*}{d\omega} > 0$.

Now, $\frac{dU^*}{d\alpha} = (\frac{\sqrt{1-\psi}}{2\sqrt{2}\sqrt{\alpha}})(\frac{4\omega[\sqrt{v+\omega}-\sqrt{v-\omega}]-[(v-\omega)\sqrt{v-\omega}+(v+\omega)\sqrt{v+\omega}]}{\sqrt{v+\omega}\sqrt{v-\omega}})$. Note that since the sign of $4\omega[\sqrt{v+\omega} - \sqrt{v-\omega}] - [(v-\omega)\sqrt{v-\omega} + (v+\omega)\sqrt{v+\omega}]$ is ambiguous, it is not possible to determine the sign of $\frac{dU^*}{d\alpha}$. This proves part (iii).

Maximization of U^i as given by (6) with respect to R_i $i = 1, 2$ when $h(R_i)$ is given by (8) yields

$$\sqrt{R_1} = \frac{\alpha(v+\omega)}{4\sqrt{K}}, \tag{A11}$$

$$\sqrt{R_2} = \frac{\alpha(v-\omega)}{4\sqrt{K}}. \tag{A12}$$

By substituting (A11) and (A12) in (10), we get

$$U^* = v + \frac{\alpha^2}{8K}(3\omega^2 - v^2). \tag{A13}$$

Hence,

$$\frac{\partial U^*}{\partial \alpha} = \frac{2\alpha}{8K}(3\omega^2 - v^2). \tag{A14}$$

It follows from (A14) that $\frac{\partial U^*}{\partial \alpha} > 0$ for $\omega > \frac{v}{\sqrt{3}}$.

Using (A11) and (A12), it follows that $R_1^* + R_2^* = \frac{\alpha^2}{8K}(v^2 + \omega^2)$. Hence $\frac{\partial(R_1^*+R_2^*)}{\partial \omega} = \frac{\alpha^2\omega}{4K} > 0$. Further, by differentiating (A13) with respect to ω, it follows that $\frac{\partial U^*}{\partial \omega} = \frac{3\alpha^2\omega}{4K} > 0$. Hence, both aggregate spending and aggregate welfare increase with ω when $h(.)$ is given by (8).

This proves (iv). ■

Proof of Lemma 2: Using (17) and (6), Player 1's net marginal benefit of resource spending at $R_1 = 0$ is $\frac{1}{2}[\alpha_1 - (\alpha_1 - \alpha_2)h(R_2)]h'(0)v - 1$. Hence for a given R_2, Player 1 will be induced to increase R_1 beyond 0 when

$$h'(0) > \frac{2}{[\alpha_1 - (\alpha_1 - \alpha_2)h(R_2)]\, v}.$$

Since $0 < h(R_2) < 1$, it follows that $\frac{2}{[\alpha_1-(\alpha_1-\alpha_2)h(R_2)]v} < \frac{2}{\alpha_2 v}$ for any $R_2 \in [0, v]$. Hence, if $h'(0) > \frac{2}{\alpha_2 v}$, $U^1 > 0$ and increasing at $R_1 = 0$ regardless of the level of R_2. Further from Assumption 1, we know that U^1 is strictly concave over $R_1 \in [0, v]$ and $U^1 < 0$ at $R_1 = v$. Given this, we can be assured that for any $R_2 \in [0, v]$ the optimal level of R_1 is strictly between 0 and v and given by the first-order condition (18).

Exactly analogous to the case of Player 1, it can be verified that if $h'(0) > \frac{2}{\alpha_2 v}$, then for any $R_1 \in [0, v]$, the optimal level of R_2 is strictly between 0 and v and given by the first-order condition (19). ■

Proof of Proposition 5 (parts (iii)–(vi)): Let player i's best response for any $R_j \in [0, v]$ be denoted as $R_i^{RF}(R_j)$ $i, j = 1, 2$ and $i \neq j$. Suppose that Assumption 1 (which stipulates monotonicity and strict concavity of $h(.)$), and the condition stipulated in Lemma 2 is satisfied. In this case, from Lemma 2 we can be sure that $R_i^{RF}(R_j)$ is strictly interior $(0 < R_i^{RF}(R_j) < v)$ and continuous over the interval $[0, v]$ $i, j = 1, 2$ and $i \neq j$. Hence,$R_1^{RF}(R_2)$ and $R_2^{RF}(R_1)$ must cross each other at least once over the space $([0, v] \times [0, v])$. Further, since $R_1^{RF}(R_2)$ is monotonic in R_2 and negatively sloped while $R_2^{RF}(R_1)$ is monotonic in R_1 and positively sloped, it follows that their crossing point is unique. Hence, there exists a unique interior Nash equilibrium to the Cournot game. Such pure strategy equilibrium will either involve $R_1^* = R_2^* = R^*$ or $R_1^* > R_2^*$ or $R_1^* < R_2^*$. This proves part (iii).

From (18) and (19), along a symmetric equilibrium $R_1^* = R_2^* = R^*$,

$$h'(R^*) = \frac{4}{[(\alpha_1 + \alpha_2)]\, v}. \tag{A15}$$

Further,

$$\frac{2}{[\alpha_1 - (\alpha_1 - \alpha_2)h(R^*)]\, v} = \frac{2}{[\alpha_2 + (\alpha_1 - \alpha_2)h(R^*)]\, v}.$$

The above equality implies that

$$h(R^*) = \frac{1}{2}. \tag{A16}$$

When (A15) and (A16) are not simultaneously satisfied, the Nash equilibrium must be asymmetric with either $R_1^* > R_2^*$ or $R_1^* < R_2^*$.

When $R_1^* > R_2^*$, strict concavity of $h(.)$, implies that $h'(R_1^*) < h'(R_2^*)$. Hence from (18) and (19), it must be the case that

$$\frac{2}{[\alpha_1 - (\alpha_1 - \alpha_2)h(R_2^*)]v} < \frac{2}{[\alpha_2 + (\alpha_1 - \alpha_2)h(R_1^*)]v} \text{ or}$$

$$1 > h(R_1^*) + h(R_2^*). \tag{A17}$$

Further, given the monotonicity of $h(.)$, it must also be the case that

$$h(R_1^*) > h(R_2^*). \tag{A18}$$

Hence along such a Nash equilibrium, both (A17) and (A18) must be satisfied. Analogously, when,$R_1^* < R_2^*$, the following conditions must hold:

$$1 < h(R_1^*) + h(R_2^*), \tag{A19}$$

$$h(R_1^*) < h(R_2^*). \tag{A20}$$

This proves part (iv).

To establish part (v), note that Player 1's win probability as given by (17) can be rearranged as: $p_1(R_1, R_2) = \frac{1}{2} + \frac{1}{2}[\alpha_1 h(R_1)(1 - h(R_2)) - \alpha_2 h(R_2)(1 - h(R_1))]$. Since $0 < \alpha_i < 1$ for $i = 1, 2$ and $0 \leq h(.) < 1$, it follows that $|\alpha_1 h(R_1)(1 - h(R_2)) - \alpha_2 h(R_2)(1 - h(R_1))| < 1$. Hence, $0 < p_1(R_1, R_2) < 1$ for any level of resources invested by either player. Exactly the same holds for $p_2(R_1, R_2)$. From this it follows that each player is assured a positive expected payoff from the contest even if she does not invest any resources to it. That is,

$$U^1(0, R_2) > 0,$$

$$U^2(R_1, 0) > 0.$$

Thus, along any Nash equilibrium

$$U^1(R_1^*, R_2^*) = \left\{\frac{1}{2} + \frac{1}{2}\left[\alpha_1 h(R_1^*) - \alpha_2 h(R_2^*) - (\alpha_1 - \alpha_2) h(R_1^*) h(R_2^*)\right]\right\} v - R_1^* > 0,$$

$$U^2(R_1^*, R_2^*) = \left\{\frac{1}{2} + \frac{1}{2}\left[\alpha_2 h(R_2^*) - \alpha_1 h(R_1^*) + (\alpha_1 - \alpha_2) h(R_1^*) h(R_2^*)\right]\right\} v - R_2^* > 0.$$

From the above two inequalities, it follows that $v > R_1^* + R_2^*$ so that there is partial rent dissipation.

To prove part (vi), suppose that Player 1 acts as the Stackelberg leader and chooses resources prior to Player 2. We evaluate Player 1's marginal incentive to invest in resources $\frac{dU^1}{dR_1}$ at the Cournot equilibrium (R_1^*, R_2^*). Since Player 1 is the Stackelberg leader it follows that

$$\frac{dU^1}{dR_1} = \frac{\partial U^1}{\partial R_1} + \frac{\partial U^1}{\partial R_2}\frac{dR_2}{dR_1}. \tag{A21}$$

At the Cournot equilibrium, $\frac{\partial U^1}{\partial R_1} = 0$ and by totally differentiating (19), we get

$$\frac{dR_2}{dR_1} = -\frac{2(\alpha_1 - \alpha_2) h'(R_1^*)}{v h''(R_2^*)(\alpha_2 + (\alpha_1 - \alpha_2) h(R_1^*))^2} = -\frac{(\alpha_1 - \alpha_2) v h'(R_1^*)(h'(R_2^*))^2}{2h''(R_2^*)}. \tag{A22}$$

From the strict concavity of $h(.)$, it follows that $\frac{dR_2}{dR_1} > 0$. Also, since $\frac{\partial U^1}{\partial R_2} = -\frac{v}{2}[\alpha_2 + (\alpha_1 - \alpha_2) h(R_1)] h'(R_2)$, using (19), it follows that at (R_1^*, R_2^*), $\frac{\partial U^1}{\partial R_2} = -1$. By substituting these values into (A21) we find that $\frac{dU^1}{dR_1} = \frac{(\alpha_1 - \alpha_2) v h'(R_1^*)(h'(R_2^*))^2}{2h''(R_2^*)} < 0$.

Hence as a Stackelberg leader, Player 1 is induced to reduce her expenditure below what she would spend at the Cournot equilibrium.

Suppose now that Player 2 acts as the Stackelberg leader. We evaluate Player 2's marginal incentive to invest in resources $\frac{dU^2}{dR_2}$ at the Cournot equilibrium (R_1^*, R_2^*). This is given by

$$\frac{dU^2}{dR_2} = \frac{\partial U^2}{\partial R_2} + \frac{\partial U^2}{\partial R_1}\frac{dR_1}{dR_2}. \tag{A23}$$

At the Cournot equilibrium, $\frac{\partial U^2}{\partial R_2} = 0$ and by totally differentiating (18), we get

$$\frac{dR_1}{dR_2} = \frac{2(\alpha_1 - \alpha_2)h'(R_2^*)}{vh''(R_1^*)[\alpha_1 - (\alpha_1 - \alpha_2)h(R_2^*)]^2} = \frac{v(\alpha_1 - \alpha_2)(h'(R_1^*))^2 h'(R_2^*)}{2h''(R_1^*)}. \tag{A24}$$

From the strict concavity of $h(.)$, it follows that $\frac{dR_1}{dR_2} < 0$. Also, using (18), it follows that $\frac{\partial U^2}{\partial R_1} = -\frac{v}{2}[\alpha_1 - (\alpha_1 - \alpha_2)h(R_2)]h'(R_1) = -1 < 0$. By substituting these values into (A23), we find that $\frac{dU^2}{dR_2} = -\frac{v(\alpha_1 - \alpha_2)(h'(R_1^*))^2 h'(R_2^*)}{2h''(R_1^*)} > 0$.

Hence as a Stackelberg leader, Player 2 is induced to increase her expenditure above what she would spend at the Cournot equilibrium. This proves part (vi). ■

Proof of Corollary 2: When (7) holds with $\psi > \frac{1}{2}$, neither condition (20) nor condition (22) can be satisfied and therefore the interior Nash equilibrium can only be of the type $R_1^* < R_2^*$. Assumption 1 and Lemma 2 ensure its existence when $h'(0) = 1 - \psi > \frac{2}{\alpha_2 v}$, that is, $\psi < 1 - \frac{2}{\alpha_2 v}$. Further, since $\frac{1}{2} < \psi < 1$, it must also be the case that $\frac{2}{\alpha_2 v} < \frac{1}{2}$ or $\alpha_2 v > 4$. ■

Proof of Proposition 6: Throughout this proof, we abbreviate $h(R_i) = h^i$, $h'(R_i) = h'^i$, and $h''(R_i) = h''^i$ for $i = 1, 2$ for ease of exposition.

Using (6), (17), and the parameterization in Assumptions 2 and 6, the first-order conditions with respect to R_1 and R_2 when $\omega = 0$ are

$$\frac{v}{2}[\alpha + \Delta(1 - 2h^2)]h'^1 - 1 = 0, \tag{A25}$$

$$\frac{v}{2}[\alpha - \Delta + 2\Delta(h^1)]h'^2 - 1 = 0. \tag{A26}$$

Using total differentiation and Cramer's rule, we get

$$\frac{dR_1^*}{d\alpha} = \frac{\begin{vmatrix} -h'^1 & -2\Delta h'^1 h'^2 \\ -h'^2 & \frac{2h''^2}{vh'^2} \end{vmatrix}}{\begin{vmatrix} \frac{2h''^1}{vh'^1} & -2\Delta h'^1 h'^2 \\ 2\Delta h'^1 h'^2 & \frac{2h''^2}{vh'^2} \end{vmatrix}}, \tag{A27}$$

$$\frac{dR_2^*}{d\alpha} = \frac{\begin{vmatrix} \frac{2h''^1}{vh'^1} & -h'^1 \\ 2\Delta h'^1 h'^2 & -h'^2 \end{vmatrix}}{\begin{vmatrix} \frac{2h''^1}{vh'^1} & -2\Delta h'^1 h'^2 \\ 2\Delta h'^1 h'^2 & \frac{2h''^2}{vh'^2} \end{vmatrix}}. \tag{A28}$$

Let $\hat{D} = \begin{vmatrix} \frac{2h''^1}{vh'^1} & -2\Delta h'^1 h'^2 \\ 2\Delta h'^1 h'^2 & \frac{2h''^2}{vh'^2} \end{vmatrix}$. From (A27) and (A28) it follows that $\frac{d(R_1^* + R_2^*)}{d\alpha} = \frac{-\frac{2h'^1 h''^2}{vh'^2} - \frac{2h'^2 h''^1}{vh'^1} - 2\Delta h'^1 h'^2 (h'^2 - h'^1)}{\hat{D}}$. From Assumption 1, it is clear that $\hat{D} > 0$. Hence, the sign of $\frac{d(R_1^* + R_2^*)}{d\alpha}$ is determined by its numerator term. In this numerator, while the first two terms are always positive due to strict concavity of $h(.)$, the sign of the last term depends on whether $h'^1 > h'^2$. When $h(.)$ is given by (7) and the condition in Corollary 2 holds (that is, $\frac{1}{2} < \psi < 1 - \frac{2}{(\alpha-\Delta)v}$ and $(\alpha - \Delta)v > 4$) we know that $R_1^* < R_2^*$. Hence by strict concavity of $h(.)$, $h'^1 > h'^2$ so that the last term in the numerator is also positive. Hence it follows that $\frac{d(R_1^*+R_2^*)}{d\alpha} > 0$. Accordingly, $\frac{dU^*}{d\alpha} = -\frac{d(R_1^*+R_2^*)}{d\alpha} < 0$. This proves part (i).

Using the first-order conditions given by (A25) and (A26) and applying total differentiation with respect to Δ and Cramer's rule, we get

$$\frac{dR_1^*}{d\Delta} = \frac{\begin{vmatrix} -h'^1(1-2h^2) & -2\Delta h'^1 h'^2 \\ -h'^2(2h^1-1) & \frac{2h''^2}{vh'^2} \end{vmatrix}}{\hat{D}}, \tag{A29}$$

$$\frac{dR_2^*}{d\Delta} = \frac{\begin{vmatrix} \frac{2h''^1}{vh'^1} & -h'^1(1-2h^2) \\ 2\Delta h'^1 h'^2 & -h'^2(2h^1-1) \end{vmatrix}}{\hat{D}}. \tag{A30}$$

From (A29) and (A30), it follows that

$$\frac{dR_1^*}{d\Delta} = \frac{-\frac{2h'^1 h''^2(1-2h^2)}{vh'^2} - 2\Delta h'^1 (h'^2)^2 (2h^1-1)}{\hat{D}},$$

$$\frac{dR_2^*}{d\Delta} = \frac{-\frac{2h'^2 h''^1(2h^1-1)}{vh'^1} + 2\Delta h'^2 (h'^1)^2 (1-2h^2)}{\hat{D}}.$$

Accordingly,

$$\frac{d(R_1^*+R_2^*)}{d\Delta} = \frac{-\frac{2h'^1 h''^2(1-2h^2)}{vh'^2} - \frac{2h'^2 h''^1(2h^1-1)}{vh'^1} - 2\Delta h'^1 (h'^2)^2 (2h^1-1) + 2\Delta h'^2 (h'^1)^2 (1-2h^2)}{\hat{D}}.$$

Recall that $\hat{D} > 0$. Hence, the sign of $\frac{d(R_1^*+R_2^*)}{d\Delta}$ is determined by its numerator term. When $h(.)$ is given by (7) and the condition in Corollary 2 holds, $\psi > \frac{1}{2}$ so that $(2h^1 - 1) > 0$ while $(1 - 2h^2) < 0$. From this it is apparent that the last two terms in the numerator are strictly negative. Further,

$$-\frac{2h'^1 h''^2(1-2h^2)}{vh'^2} - \frac{2h'^2 h''^1(2h^1-1)}{vh'^1} = \frac{4(1-\psi)(R_1^* - R_2^*)}{(R_1^*+1)^2(R_2^*+1)^2 v}.$$

Since $R_1^* < R_2^*$ when Corollary 2 holds, it follows that this expression is also negative. Hence, $\frac{d(R_1^*+R_2^*)}{d\Delta} < 0$. Accordingly, it is also the case that $\frac{dU^*}{d\Delta} = -\frac{d(R_1^*+R_2^*)}{d\Delta} > 0$. This proves part (ii). ■

Proof of Proposition 7: Since the expected payoffs are given by (6) with $p_i(R_1, R_2)$ given by (17), $h(.)$ given by (8) and parameterizations based on Assumptions 2 and 6, it can be shown that any Cournot–Nash equilibrium will be strictly interior with $R_i^* \in (0, v)$.[30] The first-order conditions for such Cournot–Nash equilibrium are given by

$$\frac{1}{4\sqrt{R_1 K}} \left\{ (\alpha + \Delta) - 2\Delta\sqrt{\frac{R_2}{K}} \right\} (v + \omega) = 1, \tag{A31}$$

[30] See Appendix part (iv) in the online Supporting Information.

$$\frac{1}{4\sqrt{R_2K}}\left\{(\alpha-\Delta)+2\Delta\sqrt{\frac{R_1}{K}}\right\}(v-\omega)=1. \tag{A32}$$

Simultaneously solving (A31) and (A32) gives us the following solution:

$$\sqrt{\frac{R_1^*}{K}}=\frac{2K(\alpha+\Delta)(v+\omega)-\Delta(\alpha-\Delta)(v+\omega)(v-\omega)}{8K^2+2\Delta^2(v+\omega)(v-\omega)}, \tag{A33}$$

$$\sqrt{\frac{R_2^*}{K}}=\frac{2K(\alpha-\Delta)(v-\omega)+\Delta(\alpha+\Delta)(v+\omega)(v-\omega)}{8K^2+2\Delta^2(v+\omega)(v-\omega)}. \tag{A34}$$

Using (A33) and (A34), it follows that along the Cournot–Nash equilibrium $R_1^*>R_2^*$ iff,

$$K>\frac{\alpha\Delta(v+\omega)(v-\omega)}{[(\alpha+\Delta)(v+\omega)-(\alpha-\Delta)(v-\omega)]}. \tag{A35}$$

Since from (8), $K>(v+\omega)$, (A35) is always satisfied so that in equilibrium, $R_1^*>R_2^*$.

This proves (i).

Suppose $\omega=0$. Then it follows from (A33) and (A34) that

$$\frac{\partial(R_1^*+R_2^*)}{\partial\alpha}=\frac{K\alpha v^2}{2[4K^2+\Delta^2v^2]}>0, \tag{A36}$$

$$\frac{\partial(R_1^*+R_2^*)}{\partial\Delta}=\frac{K\Delta v^2(2K+\alpha v)(2K-\alpha v)}{2[4K^2+\Delta^2v^2]}, \tag{A37}$$

$$U^*=v-R_1^*-R_2^* \text{ with } \frac{\partial U^*}{\partial\alpha}=-\frac{\partial(R_1^*+R_2^*)}{\partial\alpha} \text{ and } \frac{\partial U^*}{\partial\Delta}=-\frac{\partial(R_1^*+R_2^*)}{\partial\Delta}. \tag{A38}$$

(ii) follows immediately from (A36) and (A38). Also note that R.H.S. of (A37) is positive since $K>v$. This along with (A38) proves (iii).

Suppose now that $\omega>0$.

Using (A33) we get

$$\frac{\partial}{\partial\omega}\sqrt{\frac{R_1^*}{K}}=\frac{4K\left\{4K^2(\alpha+\Delta)+4K\omega\Delta(\alpha-\Delta)+\Delta^2(\alpha+\Delta)(v+\omega)^2\right\}}{\left(8K^2+2\Delta^2(v^2-\omega^2)\right)^2}. \tag{A39}$$

Note that the R.H.S. of (A39) is always positive. Hence, it follows that $\frac{\partial R_1^*}{\partial\omega}>0$.

Using (A34) we get

$$\frac{\partial}{\partial\omega}\sqrt{\frac{R_2^*}{K}}=-\frac{4K\{4K^2(\alpha-\Delta)+4K\omega\Delta(\alpha+\Delta)+\Delta^2(\alpha-\Delta)(v-\omega)^2\}}{\left(8K^2+2\Delta^2(v^2-\omega^2)\right)^2}. \tag{A40}$$

Note that the R.H.S. of (A40) is always negative. Hence it follows that $\frac{\partial R_2^*}{\partial\omega}<0$.

Since by definition, $R_i=K(\sqrt{\frac{R_i}{K}})^2$ for $i=1,2$, it follows that

$$\frac{\partial(R_1^*+R_2^*)}{\partial\omega}=2K\left(\sqrt{\frac{R_1^*}{K}}\frac{\partial}{\partial K}\left(\sqrt{\frac{R_1^*}{K}}\right)+\sqrt{\frac{R_2^*}{K}}\frac{\partial}{\partial K}\left(\sqrt{\frac{R_2^*}{K}}\right)\right). \tag{A41}$$

Hence, when ω is arbitrarily close to v, using (A33), (A34), (A39), (A40), and (A41), it follows that

$$\frac{\partial(R_1^* + R_2^*)}{\partial\omega} = \frac{v(\alpha+\Delta)(K^2(\alpha+\Delta) + Kv\Delta(\alpha-\Delta) + v^2\Delta^2(\alpha+\Delta))}{(4K^3)}. \tag{A42}$$

By inspecting (A42), it is clear that $\frac{\partial(R_1^*+R_2^*)}{\partial\omega} > 0$.

Since, $U = U^1 + U^2 = v + [(\alpha+\Delta)h^1 - (\alpha-\Delta)h^2 - 2\Delta h^1 h^2]\omega - R_1 - R_2$, it follows that

$$\frac{dU^*}{d\omega} = ((\alpha+\Delta)h^{*1} - (\alpha-\Delta)h^{*2} - 2\Delta h^{*1}h^{*2}) + \left[\frac{\omega - v}{v+\omega}\right]\frac{\partial R_1^*}{\partial\omega} - \left[\frac{v+\omega}{v-\omega}\right]\frac{\partial R_2^*}{\partial\omega}. \tag{A43}$$

Note that $(\alpha+\Delta)h^{*1} - (\alpha-\Delta)h^{*2} - 2\Delta h^{*1}h^{*2} = p_1{}^* - p_2^*$. Since $R_1^* > R_2^*$, it follows that $p_1{}^* > p_2^*$ so that the first component of (A43) is positive. Given that $\frac{\partial R_1^*}{\partial\omega} > 0$, $\frac{\partial R_2^*}{\partial\omega} < 0$, and $\omega < v$, it follows that the second component of (A43) is negative while the last component is positive.

When ω is arbitrarily close to v, the second component (which is the only component in (A43) that is negative) is approximately 0. Hence, it follows that $\frac{dU^*}{d\omega} > 0$. This completes the proof of (iv). ■

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Supporting Information

Additional Supporting Information may be found in the online version of this article at the publisher's website:

Online Appendix