A Macroeconomic Model of Equities and Real, Nominal, and Defaultable Debt

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Abstract

I show that a simple, structural New Keynesian model with Epstein-Zin preferences is consistent with a wide variety of asset pricing facts, such as the size and variability of risk premia on equities, real and nominal government bonds, and corporate bonds—the equity premium puzzle, bond premium puzzle, and credit spread puzzle, respectively. I thus show how to unify a variety of asset pricing puzzles from finance into a simple, structural framework. Conversely, I show how to bring standard macroeconomic models into agreement with a wide range of asset pricing facts.

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1. Introduction

Traditional macroeconomic models, such as Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007), ignore asset prices and risk premia and, in fact, do a notoriously poor job of matching financial market variables (e.g., Mehra and Prescott, 1985; Backus, Gregory, and Zin, 1989; Rudebusch and Swanson, 2008). At the same time, traditional finance models, such as Dai and Singleton (2003) and Fama and French (2015), ignore the real economy—even when these models use a stochastic discount factor or consumption rather than a latent factor framework, those economic variables are typically taken to be exogenous, reduced-form processes.

Yet despite this traditional separation, the linkages between the real economy and financial markets can be very important. During the 2007–09 global financial crisis and 2010–14 European sovereign debt crisis, declining asset values caused lending and the real economy to plummet, while the deteriorating economy caused private-sector risk premia to increase and asset prices to spiral further downward (e.g., Mishkin, 2011; Gorton and Metrick, 2012; Lane, 2012). These crises also led to dramatic fiscal and monetary policy interventions that were well beyond the range of past experience.[1] Reduced-form finance models that perform well based on past empirical correlations may perform very poorly when those past correlations no longer hold, such as when there is a structural break or unprecedented policy intervention as observed during these crises. A structural macroeconomic model is more robust to these breaks and can immediately provide insights into their effects on financial markets and the real economy. Structural macroeconomic models can also provide useful intuition about *why* output, inflation, and asset prices co-move in certain ways and how that comovement may change after a novel policy intervention or structural break.

In the present paper, I develop a simple, structural macroeconomic model that is consistent with a wide range of asset pricing facts, such as the size and variability of risk premia on equities and real, nominal, and defaultable debt. Thus, unlike traditional macroeconomic models, the model I present here is able to match asset prices and risk premia remarkably well. Unlike traditional finance models, the model here can provide insight into the effects of novel policy

[1] For example, the U.S. Treasury bought large equity stakes in automakers and financial institutions, and insured money market mutual funds to prevent them from "breaking the buck." The Federal Reserve purchased very large quantities of longer-term Treasury and mortgage-backed securities and gave explicit forward guidance about the likely path of the federal funds rate for years into the future. See, e.g., Mishkin (2011) and Gorton and Metrick (2012). For Europe, the European Union established the European Stability Mechanism to provide quick financial backing to member countries in need, while the European Central Bank provided large and unprecedented three-year loans to banks and announced that it would purchase large quantities of euro area members' bonds if the yields on those bonds became excessively stressed (e.g., *Wall Street Journal*, 2012; European Central Bank, 2012).

interventions and structural breaks on asset prices, and provides a unified structural explanation for the behavior of risk premia for a variety of asset classes.

The model has two essential ingredients: generalized recursive preferences, as in Epstein and Zin (1989) and Weil (1989), and nominal rigidities, as in the textbook New Keynesian models of Woodford (2003) and Galí (2008). Generalized recursive preferences allow the model to generate substantial risk premia without greatly distorting the behavior of macroeconomic aggregates, while nominal rigidities allow the model to match the behavior of inflation, nominal interest rates, and nominal assets such as Treasuries and corporate bonds.

My results have important implications for both macroeconomics and finance. For macroeconomics, I show how standard dynamic structural general equilibrium (DSGE) models can be brought into agreement with a wide variety of asset pricing facts. I thus address Cochrane's (2008) critique that a failure of macroeconomic models to match even basic asset pricing facts is a sign of fundamental flaws in those models.[2] Moreover, bringing those models into better agreement with asset prices makes it possible to use those models to study the linkages between risk premia, financial markets, and the real economy.

For finance, I unify a variety of asset pricing puzzles into a simple, structural framework. This framework can then be used to study the relationships between the different puzzles with each other and with the economy. For example, Backus, Gregory, and Zin (1989), Donaldson, Johnsen, and Mehra (1990), and Den Haan (1995) argue that the yield curve ought to slope downward on average because interest rates tend to be low during recessions, implying that bond prices are high when consumption is low (which would lead to an insurance-like, negative risk premium). According to the model here, the nominal yield curve can slope upward even if the real yield curve slopes downward if technology shocks (or other "supply" shocks) are an important source of economic fluctuations. Technology shocks cause inflation to rise when consumption falls, so that long-term nominal bonds *lose* rather than gain value in recessions, implying a positive risk premium. Similarly, the model I present here can be used to study the changes in correlations between stock and bond returns documented by Baele, Bekaert, and Inghelbrecht (2010), Campbell, Sundaram, and Viceira (2017), and others.

Previous macroeconomic models of asset prices have tended to focus exclusively on a single

[2] As Cochrane (2008) points out, asset markets are the mechansim by which marginal rates of substitution are equated to marginal rates of transformation in a macroeconomic model. If the model is wildly inconsistent with basic asset pricing facts, then by what mechanism does the model equate these marginal rates of substitution and transformation?

type of asset, such as equities (e.g., Boldrin, Christiano, and Fisher, 2001; Tallarini, 2001; Guvenen, 2009; Barillas, Hansen, and Sargent, 2009) or debt (e.g., Rudebusch and Swanson, 2008, 2012; Van Binsbergen et al., 2012; Andreasen, 2012). A disadvantage of this approach is that it's unclear whether the results in each case generalize to other asset classes. For example, Boldrin et al. (2001) show that capital immobility in a two-sector DSGE model can fit the equity premium by making the price of capital (and equity) more volatile, but this mechanism does not explain subtantial risk premia on long-term government bonds, which involve the valuation of a fixed stream of coupon payments. By focusing on multiple asset classes, I impose additional discipline on the model and ensure that its results apply more generally. Matching the behavior of a variety of assets also helps to identify model parameters, since different types of assets are relatively more informative about different aspects of the model. For example, nominal assets are helpful for identifying parameters related to inflation, and long-lived equities provide information about the persistence of shocks and household's generalized recursive preferences.

The four papers most closely related to my analysis are Tallarini (2001), Rudebusch and Swanson (2012), Campbell, Pflueger, and Viceira (2020), and Pflueger and Rinaldi (2021). Tallarini (2001) incorporates Epstin-Zin-Weil preferences into a standard real business cycle model to match the equity premium. Relative to Tallarini (2001), the model here matches nominal as well as real features of the economy, explains the behavior of multiple assets (equities and real, nominal, and defaultable debt), and works within a standard New Keynesian DSGE framework rather than a real business cycle model, which allows monetary policy to have nontrivial effects on asset prices and the economy. Rudebusch and Swanson (2012) incorporate Epstein-Zin-Weil preferences into a standard New Keynesian DSGE model to match the behavior of nominal long-term bonds.[3] In contrast to Rudebusch and Swanson (2012), the model here is substantially simpler (to clarify its essential features) and is extended to match the behavior of additional asset classes (equities, real debt, and defaultable debt). Campbell, Pflueger, and Viceira (2020, henceforth CPV) study stock and bond prices in a New Keynesian model with fixed labor and Campbell-Cochrane (1999) consumption habits. In contrast to CPV, I use Epstein-Zin-Weil (EZW) preferences rather than Campbell-Cochrane habits; EZW preferences allow households to have essentially any period utility function and any intertermporal elasticity of substitution (IES), while Campbell-Cochrane habits require a very special, highly nonlinear period utility function with an extremely low IES. Moreover, the results in CPV do not in general carry over to a model with flexible labor, since

[3] See also Van Binsbergen et al. (2012) and Andreasen (2012), who undertake similar analyses.

households with Campbell-Cochrane habits and flexible labor typically vary their hours of work endogenously to smooth consumption, so much so that risk premia are essentially eliminated (Lettau and Uhlig, 2000; Rudebusch and Swanson, 2008). Finally, Pflueger and Rinaldi (2021) address the shortcomings of CPV by considering a fully structural New Keynesian model with flexible labor, Campbell-Cochrane habits, and Greenwood, Hercowitz, and Huffman (1988) preferences; the use of GHH preferences removes households' incentive to vary labor supply in response to asset value shocks, allowing their model to match risk premia even with flexible labor. Like Pflueger and Rinaldi (2021), I work with a fully structural New Keynesian model with flexible labor; unlike those authors, I use EZW preferences rather than Campbell-Cochrane habits to match the data, which have the advantages discussed above.

A number of other papers have studied stock and bond prices jointly in a reduced-form affine framework (e.g., Eraker, 2008; Bekaert, Engstrom, and Grenadier, 2010; Lettau and Wachter, 2011; Ang and Ulrich, 2013; Koijen, Lustig, and Van Nieuwerburgh, 2013).[4] Some of these studies work with latent factors, ignoring the real economy, while others relate asset prices to the reduced-form behavior of consumption; however, none of them use a structural macroeconomic model, which has the advantages described above. Although reduced-form models sometimes fit the data better than structural macroeconomic models, this can simply be a tautological implication of Roll's (1977) critique (that any mean-variance efficient portfolio perfectly fits the mean returns of all assets), as noted by Cochrane (2008). It is only the correspondence of financial risk factors to plausible economic risks that makes reduced-form financial factors interesting.

Chen, Collin-Dufresne, and Goldstein (2009), Bhamra, Kuehn, and Strebulaev (2010), and Chen (2010) model equity and corporate bond prices jointly in an endowment economy. Those authors undertake a more detailed, structural analysis of the corporate financing decision than I consider here, but at a cost of working in a much simpler, reduced-form macroeconomic environment. In other words, I use a simple, reduced-form model of the firm in order to better focus on the structural behavior of the economy, while those authors use a simple, reduced-form model of the macroeconomy to better focus on the structural finance behavior of the firm. The advantages of the structural macroeconomic approach I take here are discussed above.

Throughout the present paper, a recurring theme is the simplicity of the model, in the interest of clarity and to help provide intuition for the underlying mechanisms. Thus, the model

[4] See also Campbell, Sundaram, and Viceira (2017), who price stocks and bonds jointly in a quadratic latent-factor framework.

here is not designed to match very detailed features of the economy or asset prices; indeed, if one pushes the model far enough, it is certain to fail at matching some features of financial markets or the macroeconomy. Thus, the model here should be viewed as a "proof of concept" that the standard New Keynesian DSGE framework can be adapted to match asset prices quite well and shows a great deal of promise for future development in this direction. The approach I take here is thus analogous to Kydland and Prescott (1982), who showed that the stochastic growth model could be extended to match key features of business cycle fluctuations; their stylized model failed to match many details of business cycles (e.g., unemployment, inflation), but provided a first step toward the equilibrium modeling of these phenomena.

The remainder of the paper proceeds as follows. In Section 2, I develop a simple New Keynesian DSGE model with Epstein-Zin preferences, show how to solve the model, and discuss the calibration of the model and its implications for macroeconomic quantities. In Section 3, I derive the prices of stocks and real, nominal, and defaultable bonds within the framework of the model, and compare the behavior of those asset prices to the data. In Section 4, I discuss a number of important features of the model, such as endogenous conditional heteroskedasticity and endogenous uncertainty vs. exogenous uncertainty shocks. Section 5 concludes. Two Appendices present all the equations of the model, discuss the numerical solution method in more detail, and provide additional figures and analysis of the basic results.

2. A Simple Macroeconomic Model

In this section, I develop a simple dynamic macroeconomic model with generalized recursive preferences and nominal rigidities. Generalized recursive preferences, as in Epstein and Zin (1989) and Weil (1989), are required for the model to match the size of risk premia in the data.[5] Nominal rigidities are necessary for the model to match the basic behavior of inflation, nominal interest rates, and nominal assets such as Treasuries and corporate bonds.

Throughout this section, I strive to keep the model as simple as possible while still matching the essential behavior of macroeconomic variables and asset prices. The goal is to maximize intuition and insight into the relationships between the macroeconomy and asset prices, and avoid tangential complications. For this reason, I deliberately follow the very simple, "textbook"

[5] See Lettau and Uhlig (2000) and Rudebusch and Swanson (2008) for a discussion of why habits in household preferences, such as Campbell and Cochrane (1999), are unable to match the size of risk premia in DSGE models with flexible labor.

New Keynesian models of Woodford (2003) and Galí (2008), extended to the case of generalized recursive preferences. In principle, more realistic, medium-scale New Keynesian models such as Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007) could also be extended to the case of Epstein-Zin preferences to achieve an even better empirical fit to the data, but at the cost of being much more complicated.

2.1 Households

Time is discrete and continues forever. There is a unit continuum of representative households, each with generalized recursive preferences as in Epstein and Zin (1989) and Weil (1989). In each period t, the representative household receives the utility flow

$$u(c_t, l_t) \equiv \log c_t - \eta \frac{l_t^{1+\chi}}{1+\chi}, \tag{1}$$

where c_t and l_t denote household consumption and labor in period t, respectively, and $\eta > 0$ and $\chi > 0$ are parameters. Note that equation (1) differs from Epstein and Zin (1989) and Weil (1989) in that period utility depends on labor as well as consumption.

The assumption of additive separability in (1) follows Woodford (2003) and Galí (2008) and simplifies many aspects of the model. For example, the household's intertemporal elasticity of substitution is unity, its Frisch elasticity of labor supply is $1/\chi$, and its stochastic discount factor (defined below) is related to c_{t+1}/c_t; without additive separability, the expressions for these quantities would all be much more complicated. The similarity of the stochastic discount factor to versions of the model without labor also facilitates comparison to the finance literature. In addition, assuming logarithmic preferences over consumption ensures that the model is consistent with balanced growth (King, Plosser, and Rebelo, 1988, 2002) and is a standard benchmark in macroeconomics (e.g., King and Rebelo, 1999).

Households can borrow and lend in a default-free one-period nominal bond market at the continuously-compounded interest rate i_t. The use of continuous compounding simplifies the bond-pricing equations below and enhances comparability to the finance literature. Each period, the household faces a flow budget constraint

$$a_{t+1} = e^{i_t} a_t + w_t l_t + d_t - P_t c_t, \tag{2}$$

where a_t denotes beginning-of-period nominal bonds, w_t and d_t denote the nominal wage and exogenous transfers to the household, respectively, and P_t denotes the price of consumption. The household faces a standard no-Ponzi-scheme constraint,

$$\lim_{T\to\infty} E_t \prod_{\tau=t}^{T} e^{-i_{\tau+1}} a_{T+1} \geq 0. \tag{3}$$

Let $(c^t, l^t) \equiv \{(c_\tau, l_\tau)\}_{\tau=t}^{\infty}$ denote a state-contingent plan for household consumption and labor from time t onward, where the explicit state-dependence of the plan is suppressed to reduce notation. Following Epstein and Zin (1989), Weil (1989), and Tallarini (2001), I assume that the household has preferences over state-contingent plans ordered by the recursive functional

$$\widetilde{V}(c^t, l^t) \;=\; (1-\beta)\, u(c_t, l_t) \;-\; \beta\alpha^{-1} \log \left[E_t \, \exp\left(-\alpha \widetilde{V}(c^{t+1}, l^{t+1})\right)\right], \tag{4}$$

where $\beta \in (0,1)$ and $\alpha \in \mathbb{R}$ are parameters, E_t denotes the mathematical expectation conditional on the state of the economy at time t, and (c^{t+1}, l^{t+1}) denotes the state-contingent plan (c^t, l^t) from date $t+1$ onward. The case $\alpha = 0$ in (4) is defined by letting $\alpha \to 0$ and corresponds to the special case of expected utility preferences. When $\alpha \neq 0$ in (4), the expectation operator is effectively "twisted" and "untwisted" by the exponential function with coefficient $-\alpha$. This leaves the household's intertemporal elasticity of substitution in (4) the same as for expected utility, but amplifies (or attenuates) the household's risk aversion with respect to gambles over future utility flows by the additional curvature parameter α, with larger values of α corresponding to greater risk aversion. Thus, generalized recursive preferences allow the household's intertemporal elasticity of substitution and coefficient of relative risk aversion to be parameterized independently. Following Hansen and Sargent (2001), the specific form of generalized recursive preferences in (4) is often referred to as "multiplier preferences".

In each period, the household maximizes (4) subject to the budget constraint (2)–(3). The state variables of the household's optimization problem are a_t and Θ_t, where the latter is a vector denoting the state of the aggregate economy at time t. The household's "generalized value function" $V(a_t; \Theta_t)$ satisfies the generalized Bellman equation

$$V(a_t; \Theta_t) \;=\; \max_{(c_t, l_t)} \; (1-\beta)\, u(c_t, l_t) \;-\; \beta\alpha^{-1} \log \left[E_t \, \exp\left(-\alpha V(a_{t+1}; \Theta_{t+1})\right)\right], \tag{5}$$

where a_{t+1} is given by (2).

It's straightforward to show (e.g., Rudebusch and Swanson, 2012), that the household's stochastic discount factor is given by

$$m_{t+1} \;\equiv\; \beta \, \frac{c_t}{c_{t+1}} \, \frac{\exp\left(-\alpha V(a_{t+1}; \Theta_{t+1})\right)}{E_t \exp\left(-\alpha V(a_{t+1}; \Theta_{t+1})\right)} \,. \tag{6}$$

Let r_t denote the one-period continuosly-compounded risk-free real interest rate. Then

$$e^{-r_t} = E_t m_{t+1}. \tag{7}$$

2.2 Firms

The economy also contains a continuum of infintely-lived monopolistically competitive firms indexed by $f \in [0,1]$, each producing a single differentiated good. Firms hire labor from households in a competitive market and have identical Cobb-Douglas production functions,

$$y_t(f) \;=\; A_t k^{1-\theta} l_t(f)^{\theta}, \tag{8}$$

where $y_t(f)$ denotes firm f's output, A_t is aggregate productivity affecting all firms, k and $l_t(f)$ denote the firm's capital and labor inputs at time t, respectively, and $\theta \in (0,1)$ is a parameter. For simplicity, following Woodford (2003) and Galí (2008), I assume that firms' capital stocks are fixed, so that labor is the only variable input to production. Intuitively, movements in the capital stock are small at business-cycle frequencies and are dominated by fluctuations in labor.[6]

Technology, A_t, follows an exogenous AR(1) process,

$$\log A_t \;=\; \rho_A \log A_{t-1} + \varepsilon_t^A, \tag{9}$$

where $\rho_A \in (-1,1]$, and ε_t^A denotes an i.i.d. white noise shock with mean zero and variance σ_A^2. For simplicity and comparability to the finance literature, I set $\rho_A = 1$ in the baseline calibration of the model, below, but consider alternative values of ρ_A as well. For simplicity and ease of exposition, I also abstract from trend technology growth—*i.e.*, the mean of $\log(A_t/A_{t-1})$ is 0.[7]

Firms set prices optimally subject to nominal rigidities in the form of Calvo (1983) price contracts, which expire with exogenous probability $1-\xi$ each period, $\xi \in [0,1)$. Each time a Calvo contract expires, the firm sets a new contract price $p_t^*(f)$ freely, which then remains in effect for the life of the new contract, with indexation to the (continuously-compounded) steady-state

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[6] Woodford (2003, p. 167) compares a model with fixed firm-specific capital to a model with endogenous capital and investment adjustment costs and finds that the basic business-cycle features of the two models are very similar. In models with endogenous capital (e.g., Christiano et al., 2005; Smets and Wouters, 2007; Altig et al., 2011), investment adjustment costs are typically included to keep the capital stock stable at higher frequencies. Thus, one can think of the fixed-capital assumption as a simple way of achieving the same result. Woodford (2003) and Altig et al. (2011) also show that firm-specific capital stocks help generate inflation persistence that is consistent with the data (see particularly Woodford, 2003, pp. 163-173).

[7] If the mean rate of technology growth is μ_A, then the firm-specific capital stocks k must also grow at rate μ_A/θ in order for the model to have balanced growth.

inflation rate $\overline{\pi}$ each period.[8] In each period $\tau \geq t$ that the contract remains in force, the firm must supply whatever output is demanded at the contract price $p_t^*(f)e^{(\tau-t)\overline{\pi}}$, hiring labor $l_\tau(f)$ from households at the market wage w_τ.

Firms are jointly owned by households and distribute all profits and losses back to households each period in an aliquot, lump-sum manner. When a firm's price contract expires, the firm chooses the new contract price $p_t^*(f)$ to maximize the value to shareholders of the firm's cash flows over the lifetime of the contract,[9]

$$\sum_{j=0}^{\infty} \xi^j E_t \Big\{ m_{t,t+j} (P_t/P_{t+j}) \big[p_{t+j}(f) y_{t+j}(f) - w_{t+j} l_{t+j}(f) \big] \;\Big|\; p_{t+j}(f) = p_t^*(f) e^{j\overline{\pi}} \Big\}, \tag{10}$$

where $m_{t,t+j} \equiv \prod_{i=1}^{j} m_{t+i}$ denotes shareholders' stochastic discount factor from period $t+j$ back to t and $y_{t+j}(f)$ and $l_{t+j}(f)$ denote the firm's output and labor in period $t+j$, respectively, conditional on the contract price $p_t^*(f)$ still being in effect.

The output of each firm f is purchased by a perfectly competitive final goods sector, which aggregates the differentiated goods into a single final good using a CES production technology,

$$Y_t \;=\; \left[\int_0^1 y_t(f)^{1/\lambda} df \right]^{\lambda}, \tag{11}$$

where Y_t denotes the quantity of the final good and $\lambda > 1$ is a parameter. Each intermediate firm f thus faces a downward-sloping demand curve for its product with elasticity $\lambda/(\lambda-1)$,

$$y_t(f) \;=\; \left(\frac{p_t(f)}{P_t} \right)^{-\lambda/(\lambda-1)} Y_t, \tag{12}$$

where $p_t(f)$ denotes the price in effect for firm f at time t (so $p_t(f) = p_\tau^*(f) e^{(t-\tau)\bar{\pi}}$, where $\tau \leq t$ denotes the most recent period in which firm f reset its contract price), and P_t is the CES aggregate price of the final good,

$$P_t \;\equiv\; \left[\int_0^1 p_t(f)^{1/(1-\lambda)} df \right]^{1-\lambda}. \tag{13}$$

Differentiating (10) with respect to $p_t^*(f)$ and setting the derivative equal to zero yields the standard New Keynesian price optimality condition,

$$p_t^*(f) \;=\; \lambda \, \frac{\sum_{j=0}^{\infty} \xi^j E_t \big\{ m_{t,t+j} (P_t/P_{t+j}) y_{t+j}(f) \mu_{t+j}(f) \;\big|\; p_{t+j}(f) = p_t^*(f) e^{j\overline{\pi}} \big\}}{\sum_{j=0}^{\infty} \xi^j E_t \big\{ m_{t,t+j} (P_t/P_{t+j}) y_{t+j}(f) e^{j\bar{\pi}} \;\big|\; p_{t+j}(f) = p_t^*(f) e^{j\overline{\pi}} \big\}}, \tag{14}$$

[8] The assumption of indexation keeps the model well-behaved with respect to changes in steady-state inflation. The continuous compounding is notationally simpler for some of the equations below.

[9] Equivalently, the firm can be viewed as choosing a state-contingent plan for prices that maximizes the value of the firm to shareholders.

where $\mu_t(f)$ denotes the nominal marginal cost for firm f at time t,

$$\mu_t(f) \;\equiv\; \frac{w_t l_t(f)}{\theta y_t(f)}\,. \tag{15}$$

That is, the firm's optimal contract price $p_t^*(f)$ is a monopolistic markup λ over a discounted weighted average of expected future marginal costs over the lifetime of the contract.[10]

2.3 Aggregate Resource Constraints and Government

Let L_t denote the aggregate quantity of labor demanded by firms,

$$L_t \;=\; \int_0^1 l_t(f)df. \tag{16}$$

Then L_t satisfies

$$Y_t \;=\; \Delta_t^{-1} A_t K^{1-\theta} L_t^{\theta}\,, \tag{17}$$

where $K = \int_0^1 k\,df = k$ denotes the aggregate capital stock and

$$\Delta_t \;\equiv\; \left[\int_0^1 \left(\frac{p_t(f)}{P_t}\right)^{\lambda/((1-\lambda)\theta)} df\right]^{\theta} \tag{18}$$

measures the cross-sectional dispersion of prices across firms. Δ_t has a minimum value of unity when $p_t(f) = P_t$ for all firms f; a greater degree of cross-sectional price dispersion increases Δ_t and reduces the economy's efficiency at producing final output.[11]

Labor market equilibrium requires that $L_t = l_t$, firms' labor demand equals the aggregate labor supplied by households. Equilibrium in the final goods market requires $Y_t = C_t$, where $C_t = c_t$ denotes aggregate consumption demanded by households. For simplicity, there are no government purchases or investment in the baseline version of the model.

Finally, there is a monetary authority that sets the one-period nominal interest rate i_t according to a Taylor (1993)-type policy rule,

$$i_t \;=\; r + \pi_t + \phi_\pi(\pi_t - \overline{\pi}) + \frac{\phi_y}{4}(y_t - \overline{y}_t), \tag{19}$$

[10] To be more precise, $p_t^*(f)$ is a weighted average of marginal costs deflated by the inflation index rate, $\mu_{t+j}(f)/e^{j\overline{\pi}}$. In addition, the weights in (14) depend on $y_{t+j}(f)$, which depend on the left-hand-side variable $p_t^*(f)$, so (14) is not a closed-form solution for $p_t^*(f)$. However, the closed-form solution for $p_t^*(f)$, reported in the Appendix, has the same form as (14).

[11] See Section 4.1 for additional discussion of price dispersion in the model.

where $r = -\log\beta$ denotes the continuously-compounded steady-state real interest rate, $\pi_t \equiv \log(P_t/P_{t-1})$ denotes the inflation rate, $\overline{\pi}$ the monetary authority's inflation target, $y_t \equiv \log Y_t$,

$$\overline{y}_t \equiv \rho_{\bar{y}}\overline{y}_{t-1} + (1-\rho_{\bar{y}})y_t \tag{20}$$

denotes a trailing moving average of log output, and $\phi_\pi, \phi_y \in \mathbb{R}$ and $\rho_{\bar{y}} \in [0,1)$ are parameters.[12] The term $(\pi_t - \overline{\pi})$ in (19) represents the deviation of inflation from policymakers' target and $(y_t - \overline{y}_t)$ is a measure of the "output gap" in the model.[13]

2.4 Solution Method

To solve the model, I first write each equation in recursive form, dividing nonstationary variables (Y_t, C_t, w_t, etc.) by A_t so that the resulting ratios have a nonstochastic steady state.[14] I then use the method of local approximation around the nonstochastic steady state, or perturbation methods, to compute a numerical solution to the model. The complete set of recursive equations that define the model and additional discussion are provided in Appendix A.

Macroeconomic models similar to the one developed above are typically solved using a first-order approximation (a linearization or log-linearization), but this solution method reduces all risk premia in the model to zero.[15] A second-order approximation to the model produces risk premia that are nonzero but constant over time (a constant function of the variance σ_A^2). In order for risk premia in the model to vary with the state of the economy, the model must be solved to at least third order around the steady state. Note that second- and third-order terms in the

[12] Note that interest rates and inflation in (19) are at quarterly rather than annual rates, so ϕ_y corresponds to the sensitivity of the annualized short-term interest rate to the output gap, as in Taylor (1993). I also exclude a lagged interest rate "smoothing" term on the right-hand side of (19) for simplicity. Rudebusch (2002) argues that the degree of federal funds rate smoothing from one quarter to the next is essentially zero, and that instead the residuals ε_t^i in the Taylor rule (19) are serially correlated.

[13] This is an empirically motivated definition of the output gap: it implies that the central bank will raise short-term nominal interest rates when output rises above its recent history and lower rates when output falls below that history, all else equal. The behavior of monetary policy is very important for the sign and size of risk premia on nominal and real bonds in the model. In order for the model to match these risk premia in the data, it's important that monetary policy act in a way that is consistent with the data. Defining the output gap to be the deviation of output from flexible-price output implies that interest rates would behave in an opposite manner, and generally would not allow the model to match empirical risk premia on nominal and real bonds.

[14] The equity price p_t^e (defined below) is normalized by A_t^ν rather than A_t, where ν denotes the degree of leverage. The value function V_t is normalized by defining $\tilde{V}_t \equiv V_t - \log A_t$.

[15] In the finance literature, it is standard to log-linearize the model and then take expectations of all variables assuming joint lognormality. This approximate solution method produces nonzero (but constant) risk premia, but effectively treats higher-order moments of the lognormal distribution on par with first-order economic terms. Standard perturbation methods (e.g., Judd, 1998; Swanson, Anderson, and Levin, 2006) explicitly relate higher-order moments of the shock distribution to the corresponding order of the state variables (so variance is a second-order term, skewness a third-order term, etc.), because their magnitudes are the same in theory.

model solution can be non-negligible as long as the model is sufficiently "curved", which is the case when risk aversion (related to the Epstein-Zin parameter α) is sufficiently large.

I compute third- and higher-order solutions of the model using the Perturbation AIM algorithm of Swanson, Anderson, and Levin (2006), which can compute general nth-order Taylor series approximate solutions to discrete-time recursive rational expectations models. The model above has two state variables (Δ_t, $\overline{y}_t$) and a single shock (ε^A_{t+1}) and thus can be solved to third order very quickly, in just a few seconds on a personal computer. To obtain greater accuracy over a wider range of values for the state variables, the model can be solved to higher order; the results reported below are for the fifth-order solution unless stated otherwise. (Results for fourth- and sixth-order solutions are very similar, suggesting that the Taylor series has essentially converged over the range of values considered for the state variables.) Aruoba et al. (2006) compare a variety of numerical solution techniques for standard macroeconomic models and find that higher-order perturbation solutions are among the most accurate globally as well as being the fastest to compute. Corhay et al. (2019) solve a New Keynesian model with Epstein-Zin preferences and high risk aversion using both global projection and third-order perturbation methods, and find that the two methods produce numerically very similar solutions. Swanson, Anderson, and Levin (2006) provide details of the algorithm and discuss the global convergence properties of nth-order Taylor series approximations.

A noteworthy feature of the nonlinear solution algorithm I use here, relative to the loglinear-lognormal approximation typically used in finance, is that second- and higher-order terms of the Taylor series display endogenous conditional heteroskedasticity. Letting x_t denote a generic state variable and ε_{t+1} a generic shock, the second-order Taylor series solution has terms of the form $x_t\varepsilon_{t+1}$, which have a one-period-ahead conditional variance that depends on the economic state x_t (that is, $\mathrm{Var}_t(x_t\varepsilon_{t+1})$ depends on x_t). Thus, even though the model's exogenous driving shocks ε^A_{t+1} are homoskedastic, the nonlinear solution algorithm I use here preserves the endogenous conditional heteroskedasticity that is naturally generated by the nonlinearities in the model.

2.5 Calibration

The model described above is meant to be illustrative rather than provide a comprehensive empirical fit to the data, so I calibrate rather than estimate its key parameters. The baseline calibration is reported in Table 1, and is meant to be standard, following along the lines of parameter values estimated by Smets and Wouters (2007), Altig et al. (2011), and Del Negro, Giannoni, and

Table 1: Parameter Values, Baseline Calibration

β	0.9923	θ	0.6	ϕ_π	0.5
χ	3	ξ	0.8	ϕ_y	0.75
η	0.545	λ	1.1	$\overline{\pi}$	0.008
RRA (R^c)	60	ρ_A	1	$\rho_{\bar{y}}$	0.9
		σ_A	0.007		
		$K/(4Y)$	2.5		

Schorfheide (2015) using quarterly U.S. data.

I set the household's discount factor, β, to .9923, implying a nonstochastic steady-state real interest rate of about 3 percent per year. Although this might seem a bit high, households' risk aversion drives the unconditional mean of the risk-free real rate down to about 2 percent in the stochastic case.

The household's logarithmic preferences over consumption imply an intertemporal elasticity of substitution of unity, which is higher than estimates based on aggregate data (e.g., Hall, 1988), but is similar to estimates using household-level data (e.g., Vissing-Jorgensen, 2002). Bansal and Yaron (2004) and Dew-Becker (2014) argue that estimates based on aggregate data are biased downward, providing further support for the value of unity used here. In addition, logarithmic preferences over consumption are a standard benchmark in macroeconomics (e.g., King and Rebelo, 1999).[16]

The calibrated value of $\chi = 3$ implies a Frisch elasticity of labor supply of 1/3, consistent with estimates in MaCurdy (1980) and Altonji (1986). I set the parameter η so as to normalize $L = 1$ in steady state.

The parameter α is calibrated to imply a coefficient of relative risk aversion $R^c = 60$ in steady state, using the closed-form expressions derived in Swanson (2018) for models with labor.[17] Although this value is high, it is a well-known byproduct of the model's simplicity:[18]

[16] My results are not sensitive to setting the IES equal to unity. For example, specifications with $u(c_t, l_t) = c_t^{1-\gamma}/(1-\gamma) - \eta l^{1+\chi}/(1+\chi)$ produce very similar results when γ is set to 0.9 or 1.1. Of course, these specifications do not satisfy balanced growth and are nonstationary in response to permanent technology shocks.

[17] Swanson (2018) derives the coefficient of relative risk aversion for generalized recursive preferences with flexible labor and arbitrary period utility function $u(c_t, l_t)$. For multiplier preferences with period utility function (1) and $l = 1$ in steady state, risk aversion is given by $R^c = \alpha + (1+(\eta/\chi))^{-1}$. In general, risk aversion is lower when labor supply can vary because the household is better able to insure itself from asset value shocks.

[18] For example, Piazzesi and Schneider (2006) estimate a value of 57, Rudebusch and Swanson (2012) a value of 110, Van Binsbergen et al. (2012), Andreasen (2012), and Campbell and Cochrane (1999) a value of about 80, and Tallarini (2001) a value of about 50. The nonstationarity of technology implied by $\rho_A = 1$ in the present paper increases the quantity of risk in the model here relative to Rudebusch and Swanson (2012), which allows for a lower coefficient of relative risk aversion than in that paper.

for example, households in the model have perfect knowledge of all the equations, parameter values, shock dynamics, and shock distributions, so the quantity of risk in the model is very small relative to the actual U.S. economy. As a result, the household's aversion to risk in the model must be correspondingly larger to fit the risk premia seen in the data. As an alternative to high risk aversion, one could increase the quantity of risk in the model, such as by introducing long-run risk as in Bansal and Yaron (2004), disaster risk as in Rietz (1988) and Barro (2006), parameter uncertainty as in Weitzman (2007), or model uncertainty as in Barillas, Hansen, and Sargent (2009). Barillas et al. (2009) in particular show that high risk aversion in an Epstein-Zin specification is isomorphic to a model in which households have low risk aversion but a moderate degree of uncertainty about the economic environment.[19]

Turning to the production side of the economy, I set the elasticity of output with respect to labor $\theta = 0.6$. I calibrate the Calvo contract parameter $\xi = 0.8$, implying an average contract duration of five quarters, consistent with the estimates in Altig et al. (2010) and Del Negro et al. (2015). I calibrate the monopolistic markup λ for intermediate goods to 1.1, consistent with the estimates in Smets and Wouters (2007) and Altig et al. (2010). The technology process A_t is calibrated to be a random walk, $\rho_A = 1$, as a baseline. The standard deviation of technology shocks, σ_A, is set to .007, as estimated by King and Rebelo (1999). The steady-state ratio of the capital stock to annualized output is calibrated to 2.5.

The response of monetary policy to inflation, ϕ_π, is set to 0.5, as in Taylor (1993, 1999). I set $\phi_y = 0.75$, between the values of 0.5 and 1 used by Taylor (1993) and Taylor (1999). I set the monetary authority's inflation target $\overline{\pi}$ to 0.8 percent per quarter, implying a nonstochastic steady-state inflation rate of about 3.2 percent per year. Although this is higher than the value of about 2 percent used by many central banks as their current official inflation target, there are two reasons why a higher number is appropriate here: First, a steady-state inflation rate of 2 percent is too low to explain the historical average level of nominal yields in the U.S. and U.K. (and many other countries), even over relatively recent samples such as 1990–2019, as I show below. Second, households' risk aversion drives the unconditional mean of inflation in the stochastic version of the model a bit below the nonstochastic steady-state value. Finally, I calibrate $\rho_{\bar{y}} = 0.9$, implying

[19] See also Campanale, Castro, and Clementi (2010), who emphasize that the quantity of consumption risk in a standard DSGE model is very small, and thus the risk aversion required to match asset prices must be correspondingly larger. The simplifying representative-household assumption can also be relaxed. Mankiw and Zeldes (1991), Parker (2001), and Malloy, Moskowitz, and Vissing-Jorgensen (2009) show that the consumption of stockholders is more volatile (and more correlated with the stock market) than the consumption of nonstockholders, so the required level of risk aversion in a representative-agent model is higher than it would be in a model that recognized that stockholders have more volatile consumption (Guvenen, 2009).

that the monetary authority uses the deviation of current output from its average level over the past roughly 2.5 years to approximate the output gap.

2.6 Impulse Response Functions

Figure 1 plots impulse response functions for the model to a one-standard-deviation (0.7 percent) positive technology shock, under the baseline calibration described above. The dashed red lines in each panel report standard impulse response functions for the first-order (log-linear) solution to the model, while the solid blue lines report impulse response functions for the nonlinear, fifth-order Taylor series solution.[20] I start by describing the linear impulse response functions (dashed red lines), and then describe how the fifth-order impulse response functions (solid blue lines) differ from their linear counterparts.

The top left panel of Figure 1 reports the impulse response function for consumption, C_t, to the shock. Consumption jumps upward on impact, as higher productivity increases the supply of output and makes households wealthier in present-value terms, increasing consumption demand. The first-order impulse response function for C_t does not jump all the way to its new long-run level on impact, however, because of the increase in the real interest rate (described shortly). Instead, consumption continues to increase gradually over time toward its new steady state.

The top right panel reports the impulse response for inflation, π_t. The higher level of productivity reduces firms' marginal costs of production, and monopolistic firms set their price equal to a constant markup λ over expected future marginal costs, whenever they are able to reset their price. Thus, inflation falls on impact (by about 0.5 percent at an annualized rate) as those firms who are able to reset their prices do so. The response of inflation is persistent, however, as firms' price contracts expire only gradually.

The nominal interest rate i_t, in the middle left panel, is set by the monetary authority as a function of output and inflation according to the policy rule (19). Interest rates respond more

[20] The impulse response functions for the fifth-order solution to the model are computed as follows: The state variables of the model are initialized to their nonstochastic steady-state values. The impulse response function is computed as the period-by-period difference between a "one-shock" and a "no-shock" (baseline) scenario. In the one-shock scenario, ε_t^A is set equal to .007 in period 1, and equal to 0 from period 2 onward. In the no-shock scenario, ε_t^A is set equal to 0 in every period. Agents in the model do not have perfect foresight, so they still act in a precautionary manner even though the realized shocks turn out to be deterministically equal to 0 from period 2 onward. In principle, this nonlinear impulse response function can vary as one varies the initial point of the simulation, or may scale nonlinearly with the size of the shock ε_t^A. In practice, however, the impulse responses in Figure 2 do not vary much with the initial point and do not display much nonlinearity in the size of the shock. For example, the fifth-order impulse response functions to a *negative* 0.7 percent technology shock, which are reported in Figure B1 in Appendix B, look very similar to the negative of the blue lines in Figure 1, although they are a bit larger in magnitude.

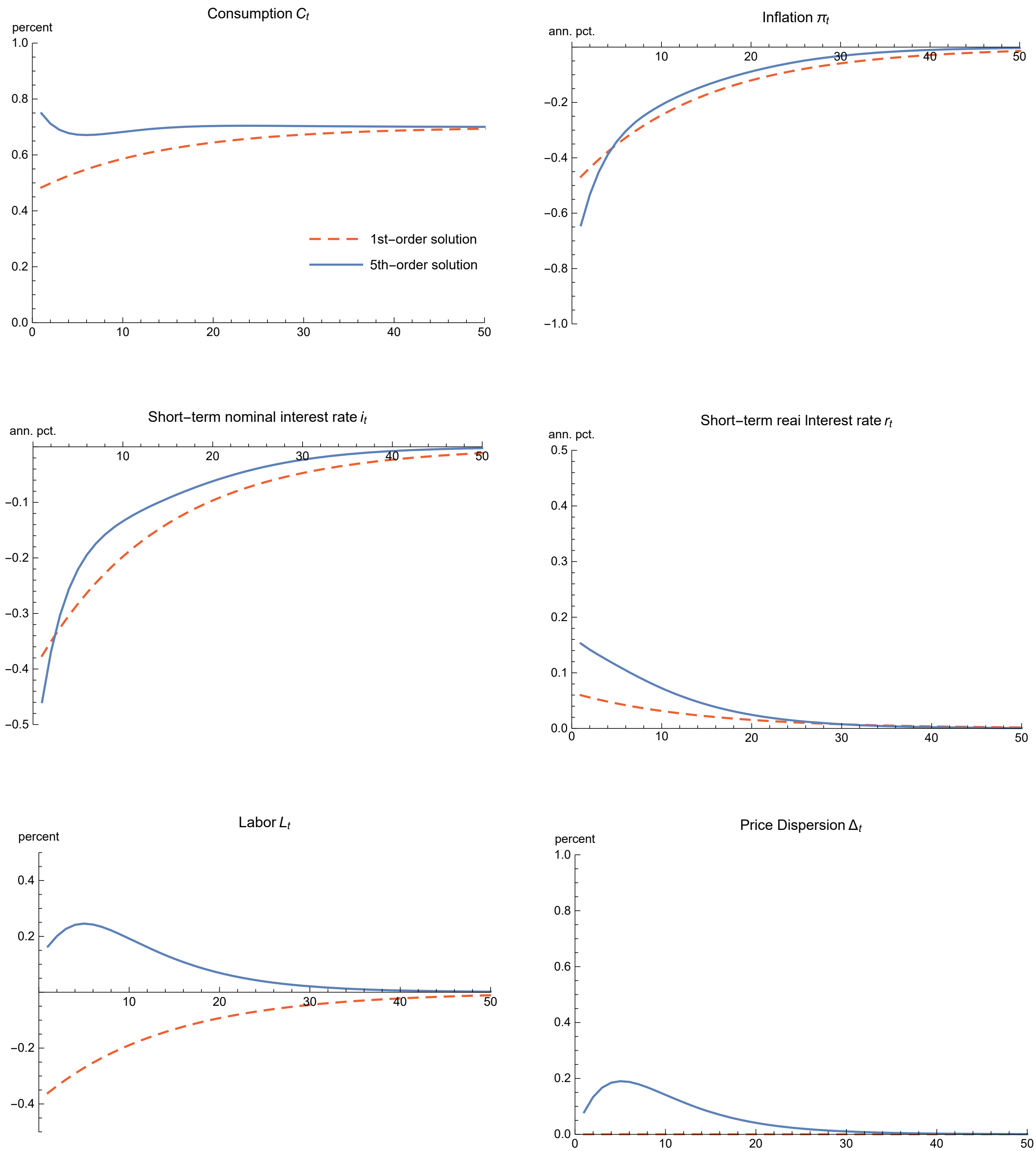

Figure 1. First-order (dashed red lines) and fifth-order (solid blue lines) impulse response functions for consumption C_t, inflation π_t, short-term nominal interest rate i_t, short-term real interest rate r_t, labor L_t, and price dispersion Δ_t to a one-standard-deviation (0.7 percent) positive technology shock in the model. See text for details.

strongly to inflation than to output, causing the nominal interest rate to decline moderately, on net, in response to the shock, about 40 basis points (at an annual rate) on impact before gradually returning to steady state. However, the nominal interest rate falls by less than inflation in response to the shock, so the real interest r_t *rises* about 5 basis points (at an annual rate) on impact, as can be seen in the middle right panel.[21] The real rate then gradually falls back to steady state.

The response of labor, L_t, is plotted in the bottom left panel. After the technology shock, households are wealthier in present value terms and want to consume more leisure; this tends to push labor downward. However, because prices are sticky and firms are monopolistic, firms hire whatever labor is necessary to satisfy output demand, which tends to push labor upward. For the simple model here, solved to first order, the former effect dominates, causing labor to decline slightly on net; indeed, this result is common in simple New Keynesian models, as pointed out by Galí (1999).[22]

However, this is no longer true for the fifth-order solution of the model, as can be seen by comparing the solid blue and red dashed lines in the bottom left panel. There are two main reasons for this difference: first, price dispersion Δ_t increases in response to the shock—as shown in the bottom right panel of Figure 1—but only for the nonlinear solution, because the linearized version of equation (18) implies shocks have no effect on Δ_t.[23] The increase in price dispersion reduces the economy's productive efficiency and increases the amount of labor required to produce any given quantity of output (see equation 17). Indeed, the hump shape in dispersion is clearly visible in the nonlinear impulse response function for labor (and to a lesser extent, consumption).[24] Second, the positive technology shock reduces the volatility of households' stochastic discount factor (SDF), for reasons discussed in detail in Section 4, below. The lower volatility of the SDF makes households effectively less risk averse and reduces their demand for precautionary savings, leading to an extra increase in consumption C_t (as can be seen in the top-left panel).

[21] Recall r_t is the *ex ante* real interest rate, so $r_t = i_t - E_t\pi_{t+1}$ to first order.

[22] In more complicated, realistic models, such as Altig et al. (2011), increased demand for investment following the technology shock is typically enough to make the increase in firms' labor demand dominate. Alternatively, a stronger monetary policy response that drives the short-term real interest rate down in response to the shock would cause consumption to jump above 0.7 percent on impact and lead to an increase in labor.

[23] The linearized version of equation (18) is $\Delta_t = \xi\Delta_{t-1}$, which implies Δ_t is invariant to shocks.

[24] The effect is essentially symmetric for a negative technology shock—i.e., Δ_t *decreases* in response to a negative technology shock (see Figure B1 in Appendix B). In the stochastic version of the model, inflation is often below the nonstochastic steady state value (due to precautionary behavior by firms, discussed below), so even if there are no shocks, $\Delta_t > 1$ will hold. Negative technology shocks can thus decrease Δ_t.

Households' greater demand for consumption requires firms to hire more labor, putting further upward pressure on L_t in the bottom-left panel.[25]

3. Asset Prices and Risk Premia

The stochastic discount factor implied by the simple macroeconomic model above can now be used to price any asset in the model. In particular, we can derive the implications of the model for the prices of equity and real, nominal, and defaultable debt.

3.1 Equity

I define an equity security in the model to be a levered claim on the aggregate consumption stream. The definition of equity as a consumption claim maximizes comparability to the finance literature and simplifies the intuition in the model; the results are very similar if equity is instead defined to be a claim on the profits of the monopolistic intermediate firm sector.[26] Each period, equity pays a dividend equal to C_t^ν, where ν is the degree of leverage. Consistent with Abel (1999) and Bansal and Yaron (2004), I calibrate $\nu = 3$. Note that ν can be interpreted as the sum of operational and financial leverage in the economy, where operational leverage results from any fixed costs of production for firms (Gourio, 2012).

Let p_t^e denote the ex-dividend price of an equity security at time t. In equilibrium,

$$p_t^e \;=\; E_t m_{t+1}(C_{t+1}^\nu + p_{t+1}^e). \tag{21}$$

Let R_{t+1}^e denote the realized gross return on equity,

$$R_{t+1}^e \;\equiv\; \frac{C_{t+1}^\nu + p_{t+1}^e}{p_t^e}\,. \tag{22}$$

I define the equity premium at time t, ψ_t^e, to be the expected excess return to holding equity for one period:

$$\psi_t^e \;\equiv\; E_t R_{t+1}^e - e^{r_t}\,. \tag{23}$$

[25] The higher level of consumption in the nonlinear case also causes the real interest rate to rise by more in the middle-right panel. The higher level of labor in the nonlinear case increases firms' marginal costs, which puts upward pressure on inflation. Inflation nevertheless falls a bit more on impact in the nonlinear case for reasons discussed in Section 4, below. The response of the nominal interest rate i_t in the nonlinear case follows in a straightforward manner from C_t and π_t, given the policy rule (19).

[26] This is because consumption, output, and monopolistic firm profits are very highly correlated in the model: $Y_t = C_t$, and firms' profits are essentially a levered claim on the output stream.

Table 2: Equity Premium as a Function of Risk Aversion and Shock Persistence

Risk aversion R^c	Shock persistence ρ_A	Equity premium ψ^e
10	1	0.62
30	1	1.96
60	1	4.19
90	1	6.70
60	.995	1.86
60	.99	1.08
60	.98	0.53
60	.95	0.17

Model-implied equity premium ψ^e, in annualized percentage points, for different values of relative risk aversion R^c and technology shock persistence ρ_A, holding the other parameters of the model fixed at their baseline values from Table 1. State variables of the model are evaluated at the nonstochastic steady state. See text for details.

Note that

$$\begin{aligned}
\psi_t^e &= \frac{E_t m_{t+1} E_t(C_{t+1}^\nu + p_{t+1}^e) - E_t m_{t+1}(C_{t+1}^\nu + p_{t+1}^e)}{p_t^e E_t m_{t+1}} \\
&= \frac{-\mathrm{Cov}_t(m_{t+1}, R_{t+1}^e)}{E_t m_{t+1}} \\
&= -\mathrm{Cov}_t\left(\frac{m_{t+1}}{E_t m_{t+1}}, R_{t+1}^e\right), \qquad (24)
\end{aligned}$$

where Cov_t denotes the covariance conditional on information at time t.[27]

The recursive equity pricing and equity premium equations (21)–(23) can be appended to the equations of the macroeconomic model in the previous section, allowing the equity premium (23) to be solved numerically along with the rest of the model. For the baseline calibration of the model in Table 1, solved to fifth order, the expected excess return on equity is about 1.05 percent per quarter (4.19 percent per year), evaluating the model's state variables at their nonstochastic steady-state values. Empirical estimates of the equity premium typically range from about 3 to 6.5 percent per year (e.g., Campbell, 1999, Fama and French, 2002), so the equity premium implied by the model is in line with the data.

The model-implied equity premium is very sensitive to both the level of risk aversion R^c and the persistence of the technology shock ρ_A. Table 2 reports values for the equity premium for several different values of R^c and ρ_A, holding the other parameters of the model fixed at their

[27] If m_{t+1} and R_{t+1}^e are jointly lognormally-distributed, as is typically assumed in the finance literature, then the equation $E_t m_{t+1} R_{t+1}^e = 1$ implies $E_t r_{t+1}^e - r_t = -\mathrm{Cov}_t(\log m_{t+1}, r_{t+1}^e) - \frac{1}{2}\mathrm{Var}_t r_{t+1}^e$, where $r_{t+1}^e \equiv \log R_{t+1}^e$. Equation (24) says essentially the same thing without assuming joint lognormality.

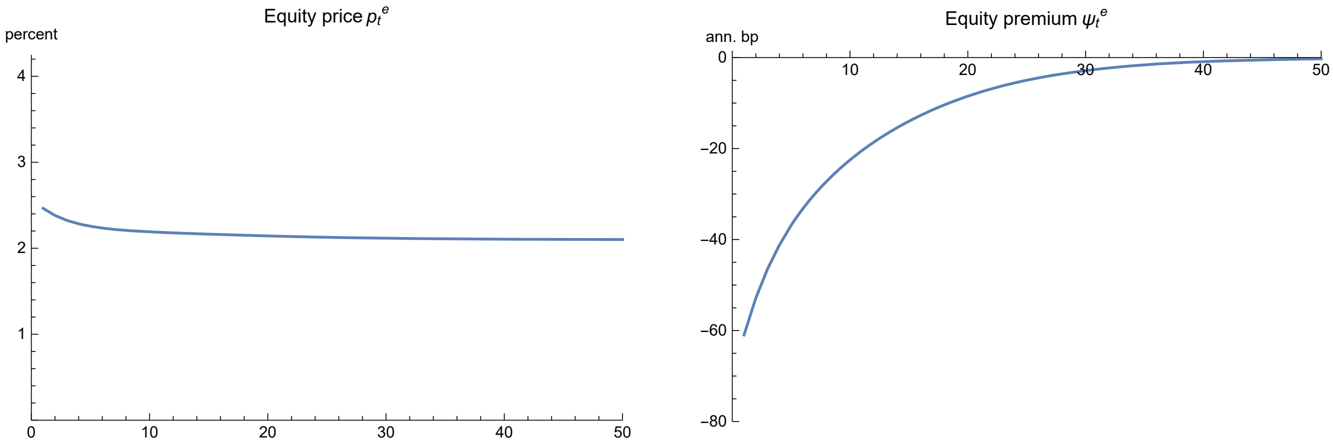

Figure 2. Nonlinear impulse response functions for the equity price p_t^e and equity premium ψ_t^e to a one-standard-deviation (0.7 percent) positive technology shock in the model, with state variables initialized to their nonstochastic steady state values. See text for details.

baseline values in Table 1. The equity premium increases about linearly along with the household's coefficient of relative risk aversion, R^c, consistent with the analysis in Swanson (2018).[28] Perhaps more surprising is the substantial drop in the equity premium for values of ρ_A that are only slightly less than unity—for example, reducing ρ_A from 1 to .995 reduces the equity premium by more than half, and reducing ρ_A from .995 to .99 cuts the equity premium almost in half again. There are two reasons why ψ^e is so sensitive to ρ_A: first, equity is very long-lived, so it is sensitive to changes in the consumption dividend even at distant horizons; second, the household's value function V_t, which enters into the stochastic discount factor (6), is also sensitive to consumption at long horizons. Reductions in ρ_A below unity have a very large effect on consumption at distant horizons, and thus significantly attenuate the response of both the equity price and the SDF to a technology shock. The subsantially lower covariance between these two variables reduces the equity premium (equation 24). (Note that the long-run risks literature, beginning with Bansal and Yaron, 2004, assumes that long-run consumption is even *more* volatile than my baseline calibration of $\rho_A = 1$; as a result, their models imply a larger equity premium than the model here, or a similar-sized equity premium with a lower degree of risk aversion.)

The equity premium in the model also varies substantially over time. Figure 2 plots the nonlinear, fifth-order impulse response functions for the equity price p_t^e and the equity premium ψ_t^e to the technology shock, computed in the same way as the nonlinear impulse response functions

[28] The equity premium increases linearly with risk aversion to second order around the nonstochastic steady state. The equity premium in Table 2 is computed to fifth order and thus is not strictly linear in risk aversion, but the intuition from the analysis in Swanson (2018) still holds.

in Figure 1. The left-hand panel of Figure 2 depicts the response of the equity price, which jumps about 2.5 percent in response to the technology shock on impact. The risk-neutral increase in the equity price would be about 2.1 percent (the leverage ratio times the increase in dividends of about 0.7 percent every period); the additional 0.4 percent increase in the price is due to the decline in the risk premium that investors require to hold the risky asset. This can be seen in the right-hand panel of Figure 2, where the equity premium drops about 60 basis points (bp) at an annual rate on impact before rising slowly back toward its initial level. Thus, the model produces an equity premium that is countercyclical, consistent with conventional wisdom in the literature (e.g., Fama and French, 1989; Campbell and Cochrane, 1999; Cooper and Priestley, 2008; Atanasov, Moller, and Priestley, 2020). The reason for this countercyclicality is that the volatility of the households' stochastic discount factor falls after a positive technology shock (and increases after a negative shock), for reasons I discuss shortly below. Over the course of a year, the standard deviation of the equity premium in the model is about 103 bp, obtained by summing the squares of the first four quarters of the impulse response and taking the square root. (Note that this is the standard deviation of the *expected* excess return on equity; I discuss the standard deviation of the ex post excess return shortly.)

To compare the model-implied time variation in the equity premium to the data, it is useful to compute the model-implied Sharpe ratio, $\psi_t^e \big/ \sqrt{\mathrm{Var}_t r_{t+1}^e}$. The average quarterly (non-annualized) Sharpe ratio in the model is $1.05/2.5 = 0.42$, compared to typical estimates of 0.2 to 0.4 in the literature (e.g., Campbell and Cochrane, 1999; Lettau and Ludvigson, 2010). The fact that the model's Sharpe ratio is at the high end of this range is not surprising, since the model is driven by a single shock and thus understates the overall volatility of equity prices; adding a monetary policy shock to the model, for example, would increase the volatility of equity without much altering its excess return (because monetary policy shocks are much less persistent than technology shocks and have only a small effect on the equity premium), and lead to a lower Sharpe ratio more in line with the middle of the range of empirical estimates.

The quarterly standard deviation of the (non-annualized) Sharpe ratio in the model is about $0.62/2.5 = 0.25$. Again, this is about in line with estimates in the literature, which range between 0.09 and 0.47 (e.g., Campbell and Cochrane, 1999; Lettau and Ludvigson, 2010, Table 11.7).

The *ex post* excess return on equity in the model has a quarterly standard deviation of about 2.5 percent per quarter, or 5 percent per year. Empirical estimates are typically in the range of 6 to 12 percent per quarter, or 12 to 24 percent per year (e.g., Campbell, 1999; Lettau and Ludvigson, 2010), so the model-implied volatility for equity returns is substantially lower than the data; however, this is again not too surprising, given that the model is driven by a single shock. Adding additional shocks to the model, such as fiscal or monetary policy shocks as in the New Keynesian DSGE literature (e.g., Smets and Wouters, 2007), would bring equity price volatility closer to the data.

From equation (24), the decline in the model-implied equity premium in Figure 2 must be due to a drop in the conditional covariance of the equity price with the stochastic discount factor. In other words, the model generates *endogenous* conditional heteroskedasticity in response to shocks, even though the exogenous technology shock is homoskedastic. This is a striking and important feature of the model. I discuss this heteroskedasticity in moe detail in Section 4, below (and see also Section 2.4, above), but the key factor is the behavior of price dispersion Δ_t. In response to a technology shock, price dispersion moves in the same direction as the shock (see the bottom-right panels of Figures 1 and B1). Because greater price dispersion reduces output and aggregate productivity (see equation 17), the response of price dispersion to a technology shock dampens the responses of the model's other variables to the shock. In addition, the sensitivity of price dispersion to a technology shock is greater if there has been a positive technology shock in the recent past—see Section 4, below. Thus, a positive technology shock today leads to a greater sensitivity of price dispersion Δ_t to future shocks, which reduces the volatility of the other variables of the model (such as consumption and the SDF) to future shocks.

The result is that the stochastic discount factor displays substantial endogenous conditional heteroskedasticity. In a perfectly homogeneous, homoskedastic model—such as the ones typically used in finance that have no labor and no nominal rigidities—there is no endogenous conditional heteroskedasticity. The only way to generate a time-varying equity premium in those models is to assume that the exogenous driving shock itself is conditionally heteroskedastic (as in Bansal and Yaron, 2004, for example).

3.2 Real and Nominal Default-Free Bonds

A default-free zero-coupon real bond in the model pays one unit of consumption at maturity. Let $\tilde{p}_t^{(n)}$ denote the nominal price of an n-period zero-coupon real bond, and $p_t^{(n)} \equiv \tilde{p}_t^{(n)}/P_t$ its real price, with $p_t^{(0)} \equiv 1$. Then for $n \geq 1$,

$$p_t^{(n)} = E_t\, m_{t+1} p_{t+1}^{(n-1)} \tag{25}$$

in equilibrium in each period t. In particular, $p_t^{(1)} = e^{-r_t}$.

A default-free zero-coupon nominal bond pays one nominal dollar at maturity. Let $p_t^{\$(n)}$ denote the nominal price of an n-period zero-coupon nominal bond, with $p_t^{\$(0)} \equiv 1$. Then for $n \geq 1$,

$$p_t^{\$(n)} \;=\; E_t\, m_{t+1} e^{-\pi_{t+1}} p_{t+1}^{\$(n-1)} \tag{26}$$

in each period t. In particular, $p_t^{\$(1)} = e^{-i_t}$.

Let $r_t^{(n)}$ denote the n-period continuously-compounded yield to maturity on a real zero-coupon bond, and $i_t^{(n)}$ the corresponding yield on an n-period nominal bond. Then

$$r_t^{(n)} \;=\; -\frac{1}{n}\, \log p_t^{(n)} \tag{27}$$

and

$$i_t^{(n)} \;=\; -\frac{1}{n}\, \log p_t^{\$(n)}. \tag{28}$$

Note that even though these bonds are free from default, they are risky in the sense that their prices can fluctuate in response to shocks, for $n > 1$.

The risk premium on a bond is typically written as a *term premium,* the difference between the yield to maturity on the bond and the hypothetical, risk-neutral yield to maturity on the same bond. The risk-neutral real price $\hat{p}_t^{(n)}$ of an n-period zero-coupon real bond is given by

$$\hat{p}_t^{(n)} \;=\; e^{-r_t} E_t\, \hat{p}_{t+1}^{(n-1)}, \tag{29}$$

where $\hat{p}_t^{(0)} \equiv 1$. The n-period real term premium $\psi_t^{(n)}$ is then

$$\psi_t^{(n)} \;\equiv\; \frac{1}{n} \left(\log \hat{p}_t^{(n)} - \log p_t^{(n)} \right) \tag{30}$$

The formula for the term premium on a nominal n-period bond, $\psi_t^{\$(n)}$, is analogous.

Note that, to first order, $\psi_t^{(n)} \approx \left(\hat{p}_t^{(n)} - p_t^{(n)}\right) / \left(n\overline{p}^{(n)}\right)$, where $\overline{p}^{(n)}$ denotes the steady-state real bond price, and

$$\begin{aligned}
\hat{p}_t^{(n)} - p_t^{(n)} \;&=\; E_t m_{t+1} E_t \hat{p}_{t+1}^{(n-1)} - E_t m_{t+1} p_{t+1}^{(n-1)} \\
&=\; -\mathrm{Cov}_t\left(m_{t+1}, p_{t+1}^{(n-1)}\right) \,+\, e^{-r_t} E_t\left(\hat{p}_{t+1}^{(n-1)} - p_{t+1}^{(n-1)}\right) \\
&=\; -E_t \sum_{j=0}^{n-1} e^{-r_{t,t+j}} \mathrm{Cov}_{t+j}\left(m_{t+j+1}, p_{t+j+1}^{(n-j-1)}\right),
\end{aligned} \tag{31}$$

where $r_{t,t+j} \equiv \sum_{\tau=t+1}^{t+j} r_\tau$ and the last line of (31) follows from forward recursion.[29] Equation (31) shows that, even though the bond price depends only on the one-period-ahead covariance between the stochastic discount factor and next period's bond price, the risk premium on the bond depends on this covariance over the entire lifetime of the bond. The term premium $\psi_t^{(n)}$ then satisfies

$$\psi_t^{(n)} \;\approx\; -\frac{1}{n\overline{p}^{(n)}} E_t \sum_{j=0}^{n-1} e^{-r_{t,t+j}} \mathrm{Cov}_{t+j}\big(m_{t+j+1}, p_{t+j+1}^{(n-j-1)}\big). \tag{32}$$

Intuitively, the term premium is larger the more negative the covariance between the SDF and the price of the bond over the lifetime of the bond.

The bond pricing and bond yield equations (25)–(30) are recursive and can be appended to the macroeconomic model described above and solved numerically along with the macroeconomic variables, equity price, and equity premium. (Note that, to consider a bond with n periods to maturity, $n-1$ bond pricing equations must be appended to the model, one for each maturity from 2 to n.)

Table 3 reports the real yield curve implied by the model, along with the corresponding average real yields estimated from inflation-indexed government bonds in the U.S. and U.K. over various sample periods. Data for U.S. inflation-indexed Treasuries (TIPS) are taken from the updated Gürkaynak, Sack, and Wright (2010) dataset on the Federal Reserve Board's web site. The first TIPS were issued in 1998, and a yield curve for maturities of 5 years or more can be estimated beginning in 1999. The first row of Table 3 thus reports average TIPS yields from 1999 to 2019, about 1.1 to 1.6 percent per year. Zero-coupon yields for shorter-maturity TIPS (down to 2 years; neither Gürkaynak et al., 2010, nor the Bank of England report zero-coupon real yields with maturity less than 2 years) can be estimated beginning in 2004, and are reported in the second row of Table 3, along with the averages for longer maturities over the same sample. This sample also excludes the period of lower TIPS liquidity in the first few years after they were introduced. Over this sample, average real yields are lower, between about 0.15 and 1.05 percent. However, the period from 2008–15 is unusual in that the financial crisis and severe recession led the Federal Reserve to reduce short-term interest rates to record lows, and the financial crisis may have led to unusually large liquidity premia in for longer-term TIPS (Fleckenstein, Longstaff, and Lustig, 2014). Thus, the third row of Table 3 reports results from 2004–19 excluding 2008–15. Over this sample, real yields average between about 0.75 and 1.3 percent.

[29] The first-order approximation in equation (31) is only used to gain intuition. To compute the numerical results in the tables and figures below, I use the nonlinear equations (25)–(30).

Table 3: Real Zero-Coupon Bond Yields, Data vs. Model

	2-yr.	3-yr.	5-yr.	7-yr.	10-yr.	(10y)−(3y)
US TIPS, 1999–2019[a]			1.11	1.34	1.59	
US TIPS, 2004–2019[a]	0.14	0.26	0.53	0.77	1.06	0.80
US TIPS, 2004–2007 ∪ 2016–2019[a]	0.74	0.83	1.01	1.16	1.32	0.39
UK indexed gilts, 1983–1995[b]	6.12	5.29	4.34		4.12	−1.17
UK indexed gilts, 1985–2019[c]		1.41	1.57	1.68	1.78	0.37
UK indexed gilts, 1990–2007 ∪ 2016–2019[c]		1.84	1.89	1.92	1.96	0.12
macroeconomic model	1.82	1.81	1.81	1.81	1.81	0.00

[a]Federal Reserve Board web site.
[b]Evans (1999).
[c]Bank of England web site.
Estimated zero-coupon real yields from inflation-indexed bonds in the U.S. and U.K., and zero-coupon real yields implied by the macroeconomic model presented above. The last column reports the difference between the 10-year and 3-year yields in each row. See text for details.

The next three rows of Table 3 report average real yields on inflation-indexed gilts in the U.K., for which we have a longer sample. Evans (1999) estimates real zero-coupon U.K. yields from 1983 to 1995, reported in the fourth row of Table 3, which average between about 4 and 6 percent over that sample. Interestingly, the real U.K. gilt yield curve slopes *downward* rather than upward over this period, by about 100–200 basis points. However, as in the U.S., the early years of the U.K. indexed gilt market may have suffered from low liquidity; thus, the fifth row of Table 3 reports average real yields from 1985 to 2019, from the Bank of England's web site. Over this longer sample, real gilt yields average about 1.4 to 1.8 percent, and the yield curve sloped upward by about 37 bp. The sixth row of Table 3 reports results for the U.K. excluding both the early years of the sample and the financial crisis and recession period. Over this sample, real yields in the U.K. are a bit higher, averaging about 1.9 percent, and the yield curve is about flat, sloping upward by 0.1 percent.

While the exact level and slope of the real yield curve depend on the sample period and country considered, the overall pattern suggests an average real yield of a bit less than 2 percent per year, with a slope that is relatively flat—neither strongly upward-sloping nor downward-sloping on average. The macroeconomic model developed in this paper fits these features well: real yields in the model average a bit less than 2 percent in the baseline calibration,[30] and the

[30]Note that the baseline value of β from Table 1 implies a real yield of a little more than 3 percent in the nonstochastic steady state. However, the real yield $r_t = 1/E_t m_{t+1}$, and $E_t m_{t+1}$ is substantially greater than $1/\beta$ in the stochastic case due to Jensen's inequality. Intuitively, households' risk aversion drives up their demand for precautionary savings in the riskless asset, lowering the risk-free rate below its nonstochastic steady-state value.

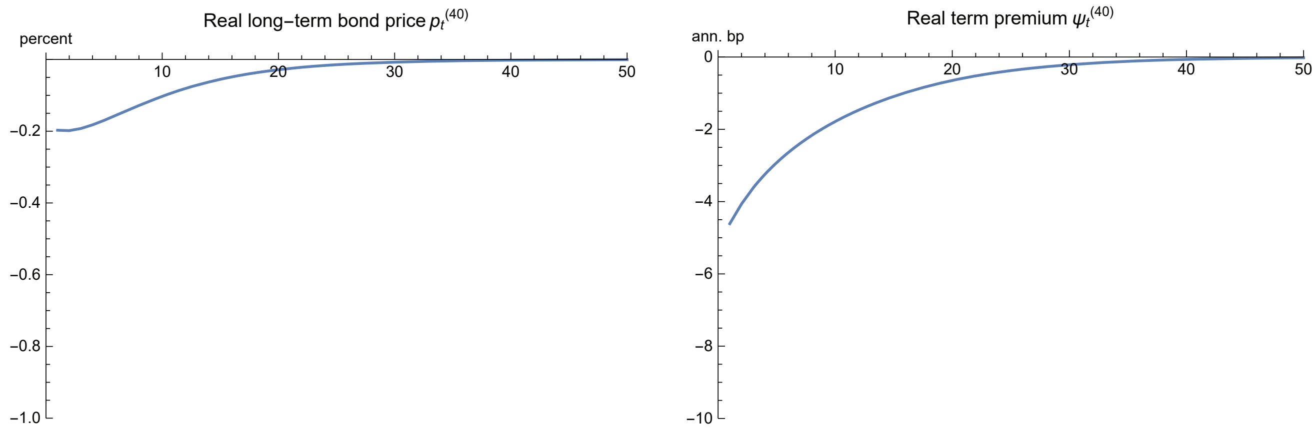

Figure 3. Nonlinear impulse response functions for real long-term bond price $p_t^{(40)}$ and real term premium $\psi_t^{(40)}$ to a one-standard-deviation (0.7 percent) positive technology shock in the model, with state variables initialized to their nonstochastic steady state values. See text for details.

real yield curve is essentially flat.

Figure 3 reports the model-implied nonlinear impulse response functions for the 10-year real bond price and term premium, computed in the same way as in Figures 1 and 2. The bond price falls only about 0.2 percent on impact, due to the small increase in short-term real rates in Figure 1, and the offsetting fall in the real term premium (right-hand panel of Figure 3). The real term premium declines in response to the shock, but by much less than the equity premium, only about 5 bp.

The small response of long-term real bond prices in Figure 3 explains why the model-implied yield curve in Table 3 is so flat on average. Intuitively, the price of real long-term bonds is fairly stable, leading to a small covariance in equation (32) and a small risk premium on average.

Table 4 compares the nominal yield curves in the model to the data. Gürkaynak, Sack, and Wright (2007) estimate zero-coupon nominal Treasury yields for the U.S. back to 1961 for maturities out to 7 years, and back to 1971 for maturities up to 10 years (updated versions of the data are available on the Federal Reserve Board's web site). From 1961 to 2019, nominal yields average about 5 to 5.9 percent. From 1971 to 2019, the average is a bit higher, about 5.1 to 6.35 percent, with an average yield curve slope of about 125 bp. However, the period from 2008–15 may not be representative because short-term nominal interest rates were constrained by the zero lower bound; the "Great Inflation" period in the 1970s and early 1980s may also be problematic since there is evidence that monetary shifted to a more anti-inflationary stance since then (e.g., Clarida, Galí, and Gertler, 2000). Thus, the third row of Table 4 reports average yields from 1990–2019, excluding 2008–15. Over this sample, nominal Treasury yields averaged about 4 to

Table 4: Nominal Zero-Coupon Bond Yields, Data vs. Model

	1-yr.	2-yr.	3-yr.	5-yr.	7-yr.	10-yr.	(10y)−(1y)
US Treasuries, 1961–2019[a]	5.02	5.24	5.42	5.70	5.91		
US Treasuries, 1971–2019[a]	5.10	5.33	5.53	5.84	6.09	6.35	1.25
US Treasuries, 1990–2007 ∪ 2016–2019[a]	4.00	4.27	4.48	4.81	5.06	5.35	1.35
UK gilts, 1970–2019[b]	6.41	6.58	6.73	6.98	7.17	7.37	0.96
UK gilts, 1990–2007 ∪ 2016–2019[b]	5.16	5.24	5.33	5.44	5.50	5.54	0.38
macroeconomic model	5.02	5.27	5.48	5.77	5.95	6.12	1.10

[a]Federal Reserve Board web site.
[b]Bank of England web site.
Empirical estimates of zero-coupon nominal yields from government bonds in the U.S. and U.K., and zero-coupon nominal yields implied by the macroeconomic model presented above. The last column reports the difference between the 10-year and 1-year yield in each row. See text for details.

5.35 percent, with a slope of about 135 bp.

The Bank of England also reports estimated zero-coupon yield curves for the U.K. going back to 1970. From 1970 to 2019, nominal gilt yields averaged about 6.4 to 7.4 percent, with a yield curve slope of about 95 bp, as reported in Table 4. Restricting attention to the period from 1990 to 2019, excluding the financial crisis, average U.K. nominal yields are somewhat lower, about 5.15 to 5.55 percent, with a slope of about 40 bp.

Again, the exact level and slope of the nominal yield curve depends on the sample period and country considered, but nominal yields seem to average about 5 to 6 percent and have an upward slope of about 100 bp. The model is able to reproduce these features of the data well: the model-implied average level of nominal yields is between about 5 and 6.1 percent, with an upward slope of 110 bp. Thus, although the model-implied real yield curve is flat, the implied nominal yield curve slopes upward substantially. As in Rudebusch and Swanson (2012), this is because technology shocks in the model make nominal bonds risky: a negative technology shock causes inflation to rise persistently at the same time that consumption falls; as a result, long-term nominal bonds in the model *lose* value in recessions. This implies that long-term nominal bonds should carry a substantial risk premium, about 106 bp over the corresponding risk-neutral yield. Thus, the simple model presented here provides a straightforward answer to the puzzle posed by Backus, Gregory, and Zin (1989), Donaldson, Johnsen, and Mehra (1990), and Den Haan (1995): namely, why does the nominal yield curve slope upward? The answer is technology shocks, or more generally, any "supply shock" that causes inflation and output to move in opposite directions, such as an oil price shock or markup shock.

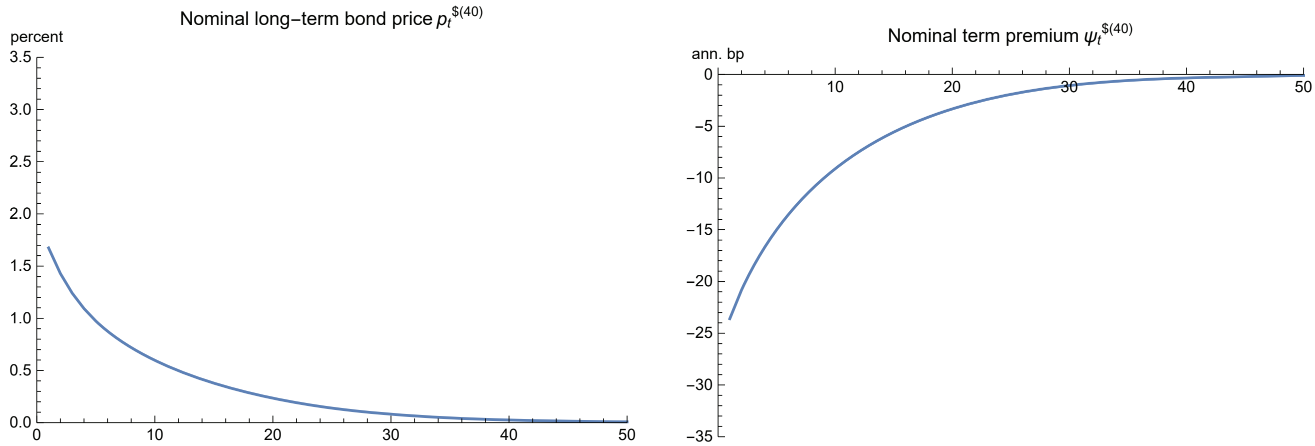

Figure 4. Nonlinear impulse response functions for nominal long-term bond price $p_t^{\$(40)}$ and term premium $\psi_t^{\$(40)}$ to a one-standard-deviation (0.7 percent) technology shock in the model, with state variables initialized to their nonstochastic steady state values. See text for details.

Of course, the larger and more important are technology or supply shocks in the model, the larger will be the term premium on nominal bonds. Thus, if supply shocks were relatively larger in the 1970s and early 1980s than in the 1960s or more recently, the model predicts that we should see a larger term premium on nominal bonds in those periods. In fact, this prediction is consistent with the data: Rudebusch, Sack, and Swanson (2007) graph several measures of the term premium—from a VAR, affine no-arbitrage models, and the Cochrane-Piazzesi (2005) "tent-shaped" factor—and for all of these measures, the estimated term premium on long-term nominal U.S. bonds is higher in the 1970s and early 1980s than in the 1960s or more recently.

Campbell, Sundaram, and Viceira (2017) also document changes in the correlation between stock and nominal bond returns over time. Although the baseline calibration of the model here has only a single shock, making it stochastically singular, extending the model to include fiscal and/or monetary policy shocks is straightforward and is standard in the medium-scale New Keynesian DSGE literature (e.g., Smets and Wouters, 2007). In these models, if the relative importance of technology or supply shocks is varied, then the correlation between stock and bond returns will vary as well. Thus, changing correlations between stock and bond returns can be mapped back to more structural features of the model.

Figure 4 plots the nonlinear impulse response functions for the 10-year nominal bond price and term premium to a one-standard-deviation positive technology shock, computed in the same way as in Figures 1–3. As discussed above, a positive technology shock causes inflation and the short-term nominal interest rate to fall (Figure 1) and the nominal long-term bond price to rise substantially (Figure 4), about 1.7 percent on impact before gradually returning back to steady state. The nominal term premium falls about 24 bp on impact, so part of the strong bond price response is due to the fall in the term premium. The reason for that fall is essentially the same as for the equity premium: the volatility of the households' stochastic discount factor declines after a positive technology shock. Thus, the model's prediction of a countercyclical term premium is consistent with conventional wisdom in the literature (e.g., Fama and French, 1989, Campbell and Cochrane, 1999, Cooper and Priestley, 2008, Piazzesi and Swanson, 2008). Over the course of a year, the standard deviation of the term premium is about 40 bp, in line with estimates from affine term structure models (e.g., Kim and Wright, 2005).

3.3 Defaultable Bonds

In the interest of simplicity, I model a defaultable bond as a depreciating nominal consol that has some probability of defaulting each period, an approach that has also been used in finance (e.g., Leland, 1994, 1998; Duffie and Lando, 2001; Chen, 2010). The *credit spread* in the model is the difference in yield between the defaultable consol and an otherwise identical consol that is free from default. I consider two cases in the analysis below: first, where the probability of default and the recovery rate given default are constant over time, and second, where those quantities vary cyclically in line with the data.

A default-free depreciating nominal consol is an infinitely-lived bond that pays a geometrically declining coupon of δ^{n-1} dollars in each period $n = 1, 2, \ldots$ after issuance. The equilibrium ex-coupon price p_t^c of the consol in period t is given by

$$p_t^c = E_t\, m_{t+1} e^{-\pi_{t+1}} (1 + \delta p_{t+1}^c), \tag{33}$$

where the size of the next coupon payment is normalized to one dollar. The very simple recursive structure of (33) makes this type of long-term bond extremely convenient to work with in a model.[31] When $\delta = 0$, the consol reduces to a one-period zero-coupon bond, and when $\delta = 1$, it behaves like a traditional nondepreciating consol. By choosing δ appropriately, the consol can be given any desired Macauley duration and made to behave very similarly to the corresponding zero-coupon bond.

[31] Leland (1994), Duffie and Lando (2001), and Chen (2010) use a nondepreciating consol to model corporate bonds, while Leland (1998) uses a depreciating consol. Rudebusch and Swanson (2008) use a (default-free) depreciating consol to study the long-term bond premium puzzle. The behavior of the depreciating consol in the simple model above and in Rudebusch and Swanson (2008) is very similar to that of a zero-coupon bond with the same Macauley duration.

The continuously-compounded yield to maturity, i_t^c, for the consol satisfies

$$p_t^c \;=\; \frac{1}{e^{i_t^c}} \;+\; \frac{\delta}{e^{2i_t^c}} \;+\; \frac{\delta^2}{e^{3i_t^c}} \;+\; \cdots\,, \tag{34}$$

implying

$$i_t^c \;=\; \log\Big(\frac{1}{p_t^c} + \delta\Big). \tag{35}$$

The Macauley duration of the consol is

$$-\frac{d\log p_t^c}{di_t^c} \;=\; 1 + \delta p_t^c. \tag{36}$$

When calibrating the model below, I set δ so that the consol has a Macauley duration of 10 years, corresponding to the approximate duration of the longer-term coupon bonds in Moody's indexes.

A defaultable consol pays a nominal coupon each period in the same way as a default-free consol, but in addition there is a chance each period that the bond will default and cease paying interest forever. In the event of default, bondholders receive a recovery rate times the previous value of the bond, which we can calibrate to the data. Thus, the defaultable consol price p_t^d satisfies

$$p_t^d \;=\; E_t\, m_{t+1} e^{-\pi_{t+1}} \Big[(1 - \mathbf{1}_{t+1}^d)(1 + \delta p_{t+1}^d) \;+\; \mathbf{1}_{t+1}^d\, \omega_{t+1}\, p_t^d\Big], \tag{37}$$

where $\mathbf{1}_t^d$ is an indicator variable equal to 1 if the bond defaults in period t and 0 otherwise, and ω_t denotes the recovery rate on the bond in the event of default. The yield to maturity i_t^d of the defaultable bond is defined by equation (35), with p_t^d in place of p_t^c, and the credit spread is the yield differential, $i_t^d - i_t^c$.

It remains to calibrate $\Pr_t\{\mathbf{1}_{t+1}^d = 1\}$ and ω_t in (37). The average rate of default for bonds initially rated Baa or BBB is about 0.6 percent per year (Moody's, 2006; Standard & Poor's, 2014), and the average recovery rate on defaulted bonds is about 42 percent (Chen, Collin-Dufresne, and Goldstein, 2009; Chen, 2010).[32] As a first calibration, I set $\Pr_t\{\mathbf{1}_{t+1}^d = 1\}$ to a constant rate of 0.15 percent per quarter and ω_t to a constant of 42 percent.

The credit spread implied by the model for this calibration is reported in the first row of Table 5. With a constant average annual default probability of 0.6 percent, the model-implied credit spread is about 34 bp. This is essentially the risk-neutral expected loss each period from

[32] The default rate on bonds *currently* rated Baa/BBB is much lower, about 0.15 percent per year on average. However, these bonds also lose value when they are downgraded, which happens with much higher probability than default. Rather than keep track of credit ratings, the probability of downgrades, and capital losses in the event of downgrade, I simply keep track of the default rate for bonds *initially* rated Baa/BBB.

Table 5: Model-Implied Credit Spread on Defaultable Bonds

average ann. default prob.	cyclicality of default prob.	average recovery rate	cyclicality of recovery rate	credit spread (bp)
.006	0	.42	0	34.0
.006	−0.3	.42	0	130.9
.006	−0.3	.42	2.5	143.1
.006	−0.15	.42	2.5	78.9
.006	−0.6	.42	2.5	367.4
.006	−0.3	.42	1.25	137.0
.006	−0.3	.42	5	155.2

Model-implied credit spread $i_t^d - i_t^c$ for defaultable vs. default-free depreciating consols with Macauley duration of 10 years. Average annualized default probability is calibrated to bonds initially rated Baa. Cyclicality of default probability and recovery rate are the loadings on the output gap, $y_t - \overline{y}_t$. See text for details.

default, (.006)(.58) = 34.8 bp, and is far less than the historical average credit spread on Baa-rated bonds of about 120 bp (e.g., Chen, Collin-Dufresne, and Goldstein, 2009; Chen, 2010).[33] Intuitively, if the risk of default in the model is uncorrelated with the stochastic discount factor, there is no additional risk premium attached to losses from default.

Empirically, however, corporate bond defaults are highly countercyclical and recovery rates highly procyclical (e.g., Chen, 2010; Giesecke, Longstaff, Schaefer, and Strebulaev, 2011; Standard & Poor's, 2011). For example, as reported by Chen (2010), the default rate averages about 0.9 percent for all bonds over the postwar period, but spikes to about 3.7 percent in 1990, 4 percent in 2001, and 5.5 percent in 2009, with smaller increases in earlier recessions (and a spike to 8.5 percent in 1933). In boom years, the default rate falls to essentially zero. Recovery rates average about 42 percent, but drop to about 20–25 percent in 1990, 2001, and 2009, and rise to 50–60 percent in boom years.

Thus, the next rows of Table 5 consider cases where the default rate, recovery rate, or both are correlated with the output gap in the model, $y_t - \overline{y}_t$. I calibrate the cyclicality of the model's annualized default rate to −0.3, which implies a drop in output of 5 percent below trend is associated with an increase in the default rate of about 1.5 percentage points.[34] While this

[33] This is the average difference between the yield on Moody's Baa and Aaa seasoned corporate bond indexes from 1921–2013. The average spread over alternative sample periods is similar. The spread between Baa-rated corporate bonds and U.S. Treasuries is even larger, about 185 bp. However, U.S. Treasuries carry an additional premium for their extreme liquidity and beneficial tax treatment, so the Baa-Aaa spread is often used instead, since Aaa corporate bonds are similar in liquidity and tax treatment to Baa-rated bonds and the probability of default on Aaa-rated bonds is still extremely low (e.g., Chen et al., 2009).

[34] To prevent the default rate in the model from becoming negative, I model it in logarithms rather than in

cyclicality is lower than in Chen (2010), my focus here is on bonds initially rated Baa/BBB, while the data in Chen (2010) is for all bonds, which includes many that were issued at ratings below investment grade. The second row of Table 5 reports the model-implied credit spread when the default rate is countercyclical, holding the recovery rate constant over time. This greatly increases the model-implied credit spread, to about 131 bp, consistent with the observed spread in the data.

The third row of Table 5 considers the case where the recovery rate is also cyclical. I calibrate the cyclicality of the recovery rate in the model to 2.5, so that a fall in output of 5 percent below trend is associated with a roughly 12.5-percentage-point decrease in the recovery rate on defaulted corporate bonds, in line with the variation reported in Chen (2010). Given this degree of cyclicality, the credit spread in the model increases a bit further, to 143 bp, still close to (and even a bit above) the value of 120 bp in the data.

In the last four rows of Table 5, I vary these cyclicality parameters to check how sensitive the credit spread is to their variation. Changes in the cyclicality of default have a large effect on the spread, while changes in the cyclicality of the recovery rate have a much smaller effect, moving the spread by only a few basis points. Intuitively, a marginal increase in the probability of default is much more costly to households because it implies an increase in the chance of a large loss; in contrast, a marginal fall in the recovery rate implies only a small chance (0.15 percent per quarter) of a modest increase in the loss. Thus, the cyclicality of recovery rates is much less important in the model and can largely be ignored.

Figure 5 reports nonlinear impulse response functions for the defaultable bond price and credit spread to a positive one-standard-deviation technology shock, computed the same way as in previous figures. The default probability and recovery rate in the model are assumed to have the same cyclicality as in the third row of Table 5, consistent with the data. On impact, the defaultable bond price jumps about 2.3 percent, in between the responses of the default-free nominal bond and equity prices in Figures 4 and 2. This is intuitive, since defaultable bonds are riskier than default-free bonds but less risky than equity in the model.

The credit spread, depicted in the right-hand panel, drops about 7 bp on impact. This is somewhat less than in the data; the standard deviation of the post-war quarterly change in the Baa-Aaa spread is about 20 bp. However, as discussed above, the model here has only one driving

levels. That is, the cyclicality of the log default rate is set to -50, which, when multiplied by the average default rate of .006 per year, produces -0.3.

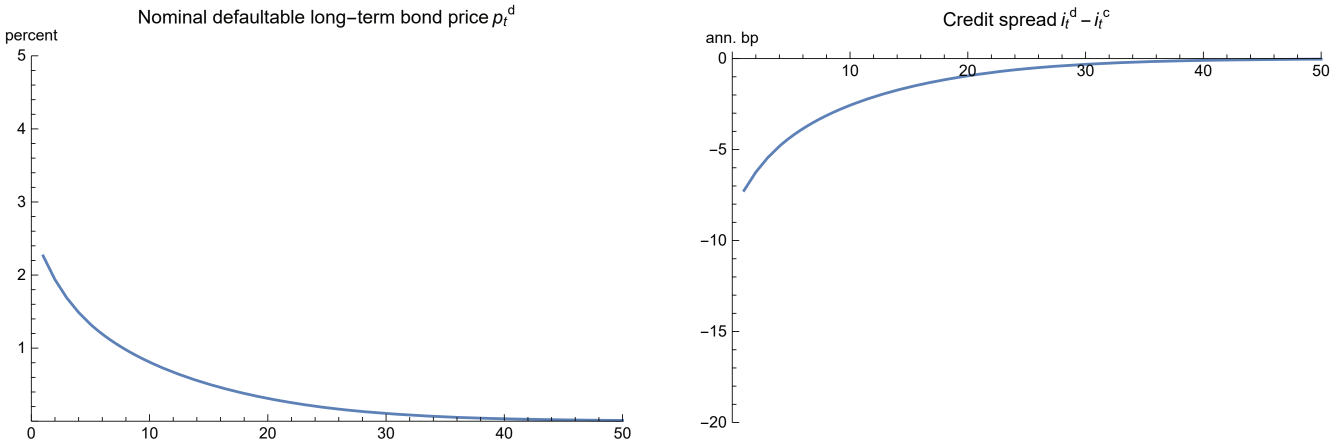

Figure 5. Nonlinear impulse response functions for defaultable long-term bond price p_t^d and credit spread $i_t^d - i_t^c$ to a one-standard-deviation (0.7 percent) technology shock in the model, with state variables initialized to their nonstochastic steady state values. See text for details.

shock; extending the model to include additional shocks would increase the overall volatility of the credit spread and bring it closer to the data.

To some extent, the model's ability to jointly fit equity returns and corporate bond yields is not surprising, since Chen, Collin-Dufresne, and Goldstein (2009), Bhamra, Kuehn, and Strebulaev (2010), and Chen (2010) achieve similar simultaneous fits in an endowment economy. Nevertheless, the present paper is the first to jointly match these data in a structural macroeconomic model. The distinction is important because results for asset prices in an endowment economy do not necessarily carry over to the case where households can smooth their consumption endogenously in response to shocks, as discussed in the Introduction. Like the present paper, Bhamra et al. (2010) and Chen (2010) use Epstein-Zin preferences, albeit with consumption and inflation taken to be exogenous, reduced-form processes. The advantage of the structural macroeconomic approach I take here is its greater robustness to structural breaks and novel policy interventions, as discussed in the Introduction. On the other hand, the much simpler macroeconomic structure in Bhamra et al. (2010) and Chen (2010) allows them to perform a more structural analysis of firms' corporate financing and default decisions. In other words, I have adopted a simple, reduced-form model of the firm in order to better focus on the structural behavior of the macroeconomy, while Bhamra et al. (2010) and Chen (2010) have adopted a simple, reduced-form model of the macroeconomy to better focus on the structural finance behavior of the firm.

4. Discussion and Extensions

The macroeconomic model developed above is essentially a "proof of concept" that a standard

New Keynesian model with Epstein-Zin preferences is consistent with a wide variety of asset pricing facts. I now discuss several features of the model in greater detail. First, I examine how the model produces endogenous conditional heteroskedasticity in response to homoskedastic exogenous shocks. Second, I discuss the relationship between the conditional heteroskedasticity in the model and the literature on "uncertainty shocks" (e.g., Bloom, 2009); I also compare the model's unitary intertemporal elasticity of substitution to the typical assumption that the IES > 1 in the long-run risks literature. Third, I extend the model to include additional shocks, such as a monetary policy shock and a fiscal policy shock, and discuss how this affects the results. Finally, I discuss the implications of the model's ability to price assets endogenously for financial frictions models, such as Bernanke, Gertler, and Gilchrist (1999), Kiyotaki and Moore (1997), and others.

4.1 Endogenous Conditional Heteroskedasticity

The macroeconomic model developed above displays endogenous conditional heteroskedasticity in response to shocks, as mentioned in Sections 2.4 and 3.1, above. Indeed, this feature of the model is crucial for generating time-varying risk premia: If the stochastic discount factor and an asset return are both homoskedastic, then the risk premium on the asset must be constant over time, as could be seen in equation (24). The fact that the model produces time-varying risk premia in Figures 2 through 5 is therefore evidence that the model-implied SDF is heteroskedastic.

In Figure 6, I analyze this heteroskedasticity in more detail. The SDF in the model is given by $m_{t+1} = \beta(C_{t+1}/C_t)^{-1}\big[\exp(-\alpha V_{t+1})/E_t \exp(-\alpha V_{t+1})\big]$, so its conditional volatility, $\mathrm{Var}_t \log m_{t+1}$, can be decomposed into two parts: the variance of consumption growth, $\mathrm{Var}_t \log(C_{t+1}/C_t)^{-1}$, and the variance of the Epstein-Zin component, $\mathrm{Var}_t \log\big[\exp(-\alpha V_{t+1})/ E_t \exp(-\alpha V_{t+1})\big]$. I report nonlinear impulse response functions for each of these two components in the left- and right-hand panels of Figure 6.[35]

In response to a positive, one-standard-deviation technology shock, the conditional variance of the SDF falls about 50 percent. The conditional variance of consumption growth, in the left-hand panel of Figure 6, falls about 40 percent, while the conditional variance of the Epstein-Zin

[35] The nonlinear impulse response functions are computed in the same way as in previous figures. I compute the conditional variance of a variable X_{t+1} in the model by defining $\mu_t^X \equiv E_t X_{t+1}$ and then $V_t^X \equiv E_t(X_{t+1} - \mu_t^X)^2$. I append these recursive equations to the rest of the model and solve them nonlinearly along with the other model variables as described earlier. Note that the conditional variance V_t^X is linearized and not log-linearized around the nonstochastic steady state because it equals zero at that point (when the variance of the technology shock σ_A^2 is set to zero). I report the change in variances V_t^X in Figure 6 in percentage terms by dividing the impulse responses (in levels) by a constant; namely, the variance V_t^X solved to fifth order and evaluated with each state variable at the nonstochastic steady state, but with the variance of the technology shock σ_A^2 set to $.007^2$.

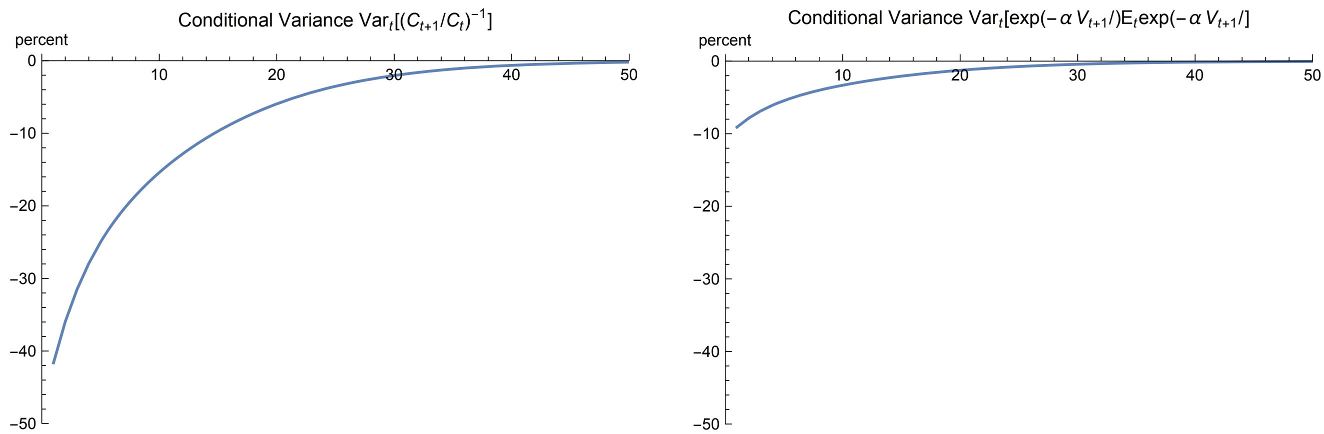

Figure 6. Nonlinear impulse response functions for conditional variances $\mathrm{Var}_t(C_{t+1}/C_t)^{-1}$ and $\mathrm{Var}_t\big[\exp(-\alpha V_{t+1})/E_t \exp(-\alpha V_{t+1})\big]$ to a one-standard-deviation (0.7 percent) positive technology shock in the model, with state variables initialized to their nonstochastic steady-state values. Impulse responses are in percentage deviation from steady state. The model-implied stochastic discount factor is $m_{t+1} = \beta(C_{t+1}/C_t)^{-1}\big[\exp(-\alpha V_{t+1})/E_t \exp(-\alpha V_{t+1})\big]$, so the two panels decompose the response of the conditional variance of the SDF to the shock. See text for details.

term (in the right-hand panel) falls about 10 percent, so the decline in SDF volatility is primarily driven by the fall in next-period consumption volatility.[36] Nevertheless, the Epstein-Zin term is still extremely important because its average *level* of volatility is so much higher than that of consumption growth—about 0.47^2 vs. 0.0065^2. The high average level of the Epstein-Zin term's volatility is what makes the decline in consumption growth volatility have a quantitatively important effect on risk premia in the model.

In Figure 7, I investigate what drives the decline in consumption volatility in Figure 6. The left column reports nonlinear impulse response functions for a one-standard-deviation (0.7 percent) positive technology shock, while the right column reports the analogous impulse response functions for a *negative* one-standard-deviation shock (−0.7 percent). In each panel, two lines are plotted: The dashed blue line is the standard nonlinear impulse response function computed in the same way as in previous figures (see footnote 23); that is, the period-by-period difference between a "one shock" and a "no shock" (baseline) scenario, with all state variables of the model initialized to their nonstochastic steady-state values. The solid green line in each panel is the period-by-period difference starting from a *different* initial point: instead of the nonstochastic steady state, the impulse responses are computed starting from the point immediately after a positive 0.7 percent technology shock in the previous period. Thus, the solid green lines depict

[36] As with previous figures, the impulse responses in Figure 6 are essentially symmetric for a negative technology shock. That is, a negative technology shock causes the conditional volatility of the SDF to increase, with a similar magnitude to (in fact, slightly larger magnitude than) Figure 6.

(a) Impulse Responses to .007 Shock ε_t^A in Period 1 (b) Impulse Responses to $-.007$ Shock ε_t^A in Period 1

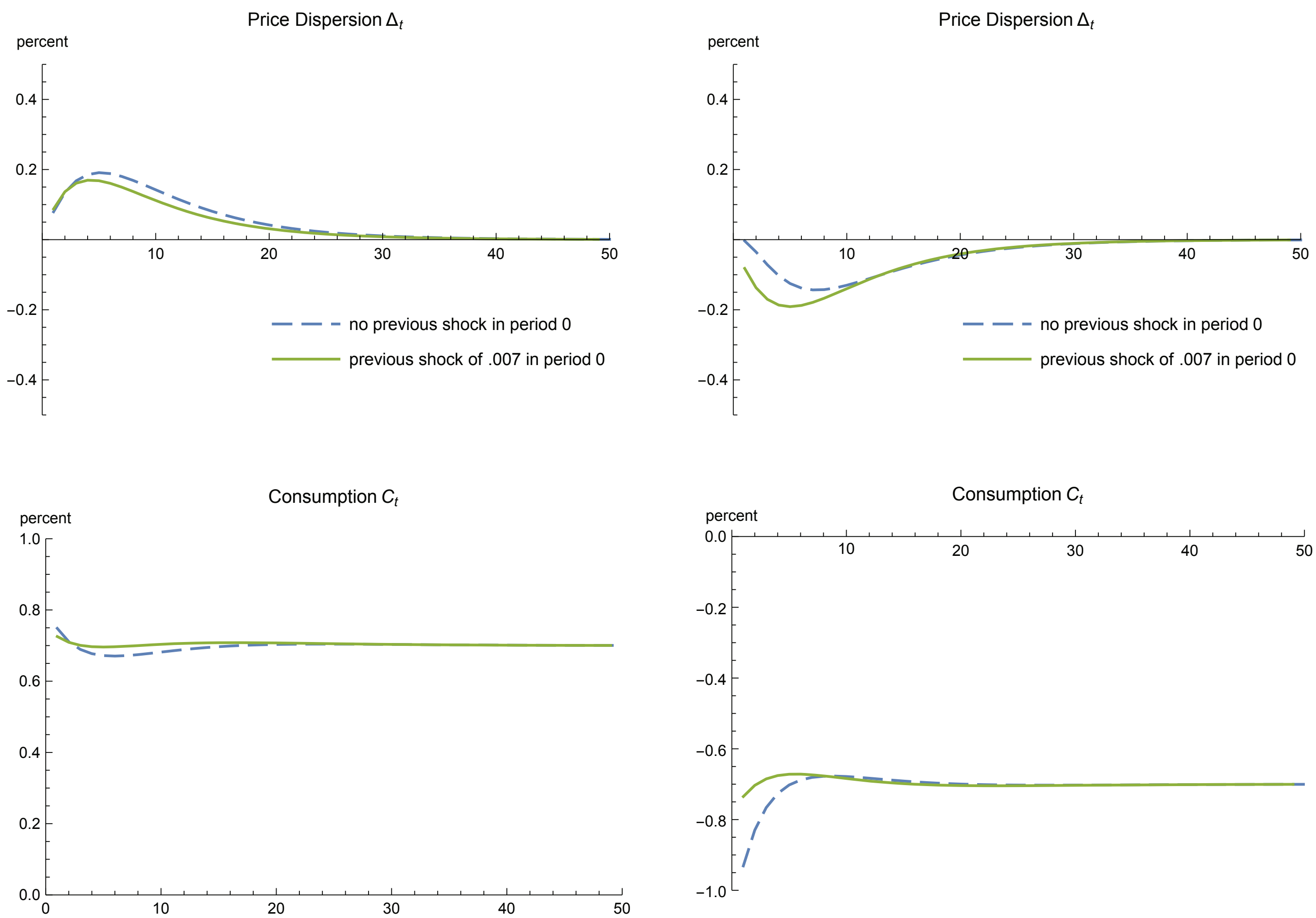

Figure 7. Comparison of nonlinear impulse response functions for price dispersion Δ_t and consumption C_t to a one-standard-deviation (0.7 percent) (a) positive vs. (b) negative technology shock in period 1. Dashed blue lines depict impulse response functions relative to a baseline of no previous shocks, with state variables initialized to their nonstochastic steady-state values, as in previous figures. Solid green lines depict impulse response functions relative to a different baseline, after a positive 0.7 percent technology shock in period 0. The figure shows that the conditional volatility of consumption is lower after a positive 0.7 percent technology shock in period 0, particularly in response to negative shocks in period 1. See text for details.

the difference between a "two shock" and a "one shock" scenario.[37]

The lower conditional volatility of consumption after a positive technology shock can be seen clearly in the bottom panels of Figure 7, particularly the bottom-right panel. There, consumption falls substantially less in response to a negative technology shock (in period 1) if that shock was

[37] This is computed as follows: in period -1, the state variables of the model are initialized to their nonstochastic steady-state values. In the "one shock" scenario, ε_t^A is set equal to .007 in period 0, and set equal to 0 from period 1 onward. In the "two shock" scenario, ε_t^A is set equal to .007 in period 0, to .007 in period 1 (for the left-hand column of Figure 7, or to $-.007$ in period 1 for the right-hand column of Figure 7), and then set equal to 0 from period 2 onward. The impulse response function is computed as the period-by-period difference between the "two-shock" scenario and the "one shock" scenario, beginning in period 1.

preceded by a positive technology shock the period before (in period 0)—that is, the solid green line does not fall by as much as the dashed blue line. In the bottom-left panel, the response of consumption to a positive technology shock (in period 1) is fairly similar whether or not there was a positive technology shock in the previous period (period 0).[38]

This attenuated behavior of consumption is driven by the response of price dispersion, Δ_t, to the technology shock, reported in the top row of Figure 7. Note first how price dispersion tends to offset the effects of the technology shock: for example, after a negative technology shock in the right-hand column of Figure 7, price dispersion falls, which tends to increase output, all else equal (equation 17). Although the net effect of the shock on output and consumption is still negative (bottom-right panel of Figure 7), the change in price dispersion moderates the effect of the technology shock. This is true for a positive technology shock in the left-hand column as well. Importantly, the effect of the technology shock on price dispersion is larger when prices are more distorted to begin with. For example, in the top-right panel of Figure 7, the solid green line falls *more* than the dashed blue line in response to the -0.7 percent shock. In the solid green line simulation, prices are more distorted to begin with because of the positive technology shock that hit the economy the period before. Thus, after a positive technology shock, the moderating effects of price dispersion in the model are *greater*, causing the volatility of output and consumption to decrease. In other words, a positive technology shock leads to a lower conditional volatility of consumption.

Figure 8 provides intuition for why Δ_t offsets the effect of the technology shock and why the response of Δ_t to a shock is larger when Δ_t itself is larger. First, note that $\Delta_t \geq 1$, and $\log \Delta_t$ is a convex function of $\log(p_t^*/P_t)$ in a neighborhood of the nonstochastic steady state, as in the figure.[39] If the economy is at the nonstochastic steady state and there are no shocks or uncertainty, then firms find it optimal to set $p_t^* = P_t$ in period t. This corresponds to point A in the figure. Even if there is uncertainty, firms still find point A optimal in the linearized version of the model, because at this point, firms' expected profits over the lifetime of the price contract

[38] As in previous figures, the effects are essentially symmetric: if the economy is hit by a negative technology shock in period 0, then the conditional volatility in Figure 6 increases rather than decreases, and consumption in the bottom-right panel of Figure 7 is relatively lower after the negative technology shock—i.e., the solid green line lies below the dashed blue line. The lower volatility of consumption growth in the bottom panels of Figure 7 is typical for other sizes of shocks as well.

[39] $\Delta_t \geq 1$ follows from Jensen's inequality: because $\lambda > 1$ and $\theta \in (0,1)$, the function $x^{\lambda/\theta}$ is convex; thus $\left(\int_0^1 x(f)^{\lambda/\theta} df\right)^{\theta/\lambda} \geq \int_0^1 x(f) df$; letting $x(f) \equiv (p_t^*(f)/P_t)^{1/(1-\lambda)}$ gives the result. Moreover, $\Delta_t = 1$ if and only if $p_t^*(f)/P_t = 1$ for almost all f. Starting from $\Delta_{t-1} = 1$, $p_t^*(f)/P_t = 1$ attains the minimum value for Δ_t, implying $\log \Delta_t$ is convex in $\log(p_t^*(f)/P_t)$ around 0.

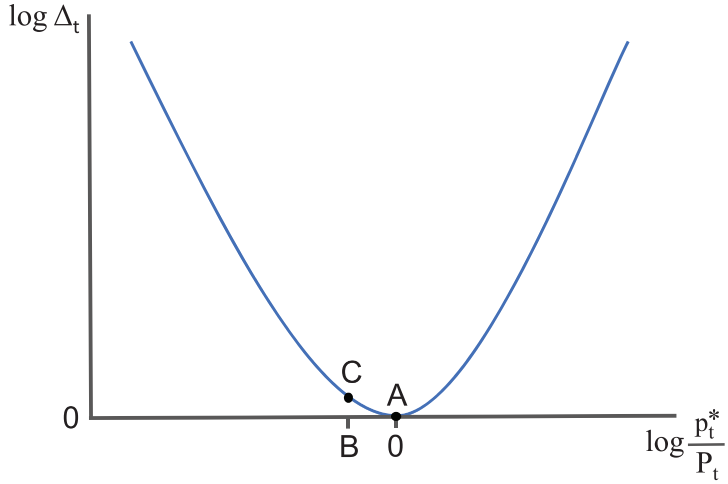

Figure 8. Illustrative graph of price dispersion Δ_t as a function of monopolistic firms' time-t real reset price, p_t^*/P_t. Point A denotes firms' risk-neutral optimal reset price, which is also the optimal reset price in the linearized model; point B denotes firms' risk-averse optimal reset price; which leads to an economywide equilibrium like point C. See text for details.

are maximized, as in equation (14). In recessions, firms will lose a bit of profit because output is too low, and in expansions firms will lose a bit of profit because output (and hence marginal cost) is too high, but in expectation the firms' price strikes an optimal balance between these two.

Now, when firms' owners (the households) are risk averse, as in the nonlinear version of the model, recessions are more painful than expansions, so it is optimal for firms to put more weight on generating profit in recessions. This causes the firms' optimal reset price to be *lower* than in the risk-neutral case, so as to generate more output and profit in the event of a recession. Thus, the risk-averse firms' optimal reset price in Figure 8 lies at a point like B, to the left of A. Of course, when all firms act this way, the equilibrium in the economy is at a point like C, where $\log \Delta_t > 0$ due to firms consistently resetting prices a bit below what the nonstochastic trend rate of inflation $\overline{\pi}$ would imply. Note that at point C, the derivative $\partial \log \Delta_t / \partial \log(p_t^*/P_t) < 0$. Thus, positive technology shocks—which cause firms' marginal costs to fall and lower p_t^*/P_t—raise Δ_t. Firms are already setting prices too low from an aggregate efficiency standpoint, and the positive technology shock exacerbates this inefficiency. Similarly, negative technology shocks lower Δ_t. This explains the impulse response functions for Δ_t in Figure 7, and why they tend to offset the effects of the technology shock.

To see why this effect is larger when price dispersion is greater, notice that $\log \Delta_t$ in Figure 8 is a convex function of $\log(p_t^*/P_t)$. If Δ_t increases from point C, this implies moving further up the curve to the left. At this point, the slope of the curve is more negative. Thus, technology shocks lead to larger changes in Δ_t when Δ_t is larger to begin with, consistent with the results in Figure 7.

It is because of this effect that the conditional volatility of consumption growth and the SDF fall after a positive technology shock, as in Figure 6. In turn, this conditional heteroskedasticity of the SDF is what causes risk premia in the model to behave countercyclically.

4.2 Uncertainty Shocks and the Intertemporal Elasticity of Substitution

In the long-run risks literature, such as Bansal and Yaron (2004), it is standard to assume that the intertemporal elasticity of substitution is substantially greater than unity. There are two reasons for that calibration: an IES > 1 guarantees that a positive shock to consumption causes stock prices to rise, and an IES > 1 ensures that an exogenous increase in volatility causes stock prices to fall.

However, the assumption of an IES > 1 is not strictly necessary for these two criteria to be satisfied, even in Bansal and Yaron (2004, henceforth BY)'s model. For example, when equity represents a levered rather than unlevered consumption claim, then equity prices in BY rise in response to a positive consumption shock if and only if the IES $> 1/\nu$, where ν is the degree of leverage. For a volatility shock, stock prices respond negatively if and only if the IES $> 1/\gamma$, where γ is the household's relative risk aversion.[40] Since $\nu, \gamma > 1$, the IES can be less than unity and still satisfy both of these criteria.

Of course, the model in the present paper differs in many respects from BY. Nevertheless, a positive shock to technology (and consumption) in the model here causes stock prices to rise, as can be seen clearly in Figure 2, even though the IES $= 1$.

To investigate the second criterion—that an increase in volatility causes stock prices to fall—I extend the model to include exogenous stochastic volatility. In particular, let the standard deviation of the technology shock each period, $\sigma_{A,t}$, follow the autoregressive process

$$\log \sigma_{A,t} \;=\; (1-\rho_\sigma) \log \bar{\sigma}_A \;+\; \rho_\sigma \log \sigma_{A,t-1} \;+\; \varepsilon_t^\sigma, \tag{38}$$

where $\bar{\sigma}_A = .007$, as in Table 1. Following Bansal and Yaron (2004), I calibrate $\rho_\sigma = 0.98$ and $\mathrm{Var}(\varepsilon_t^\sigma) = (0.1)^2$.[41]

[40] In Bansal and Yaron (2004), the coefficient A_2 for the unlevered consumption claim requires $\theta < 0$, where $\theta = (1-\gamma)/(1-1/\psi)$ and γ denotes risk aversion and ψ the IES in their paper. For the levered consumption claim, however, the coefficient $A_{2,m}$ requires $\theta/(1-\theta) < 0$ (see their equation A20), which holds if either $\theta < 0$ or $\theta > 1$. Given $\gamma > 1$, then $\theta > 1$ if and only if $\psi > 1/\gamma$.

[41] Bansal and Yaron (2004) assume a more complicated (square-root rather than logarithmic) process for $\sigma_{A,t}$ than (38), but the magnitudes in (38) are essentially the same as theirs.

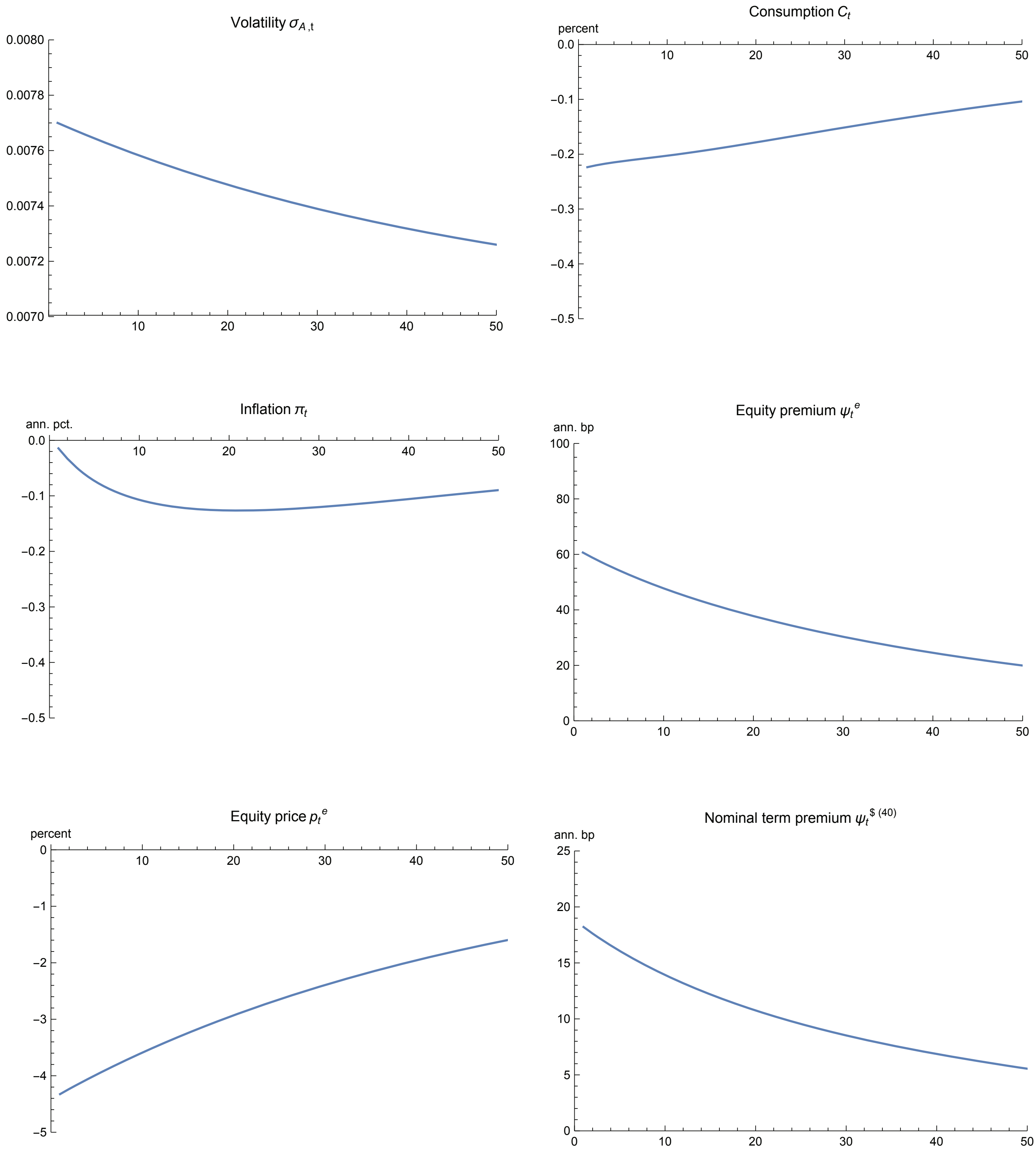

Figure 9. Nonlinear impulse response functions for volatility σ_t^A, consumption C_t, inflation π_t, the equity premium ψ_t^e, equity price p_t^e, and nominal term premium $\psi_t^{\$(40)}$ to a positive one-standard-deviation (10 percent, or .0007) volatility shock in the extended model. For clarity, the impulse response for σ_t^A is graphed as returning to its baseline value of .007 rather than a baseline of 0. See text for details.

Figure 9 plots the extended model's nonlinear impulse response functions (computed the same way as in previous figures) to a positive one-standard-deviation shock to ε_t^σ. Volatility σ_t^A increases to about .0077 on impact and slowly declines back toward its initial level of .007. Consumption drops about 0.2 percent on impact, as households increase precautionary savings, and inflation falls gradually by about 0.1 percent in response to the decrease in demand. The increase in the conditional volatility of consumption increases the volatility of the stochastic discount factor, which causes a large, 60 bp jump in the equity premium (and a large, 25 bp increase in the nominal term premium). The large and persistent rise in the equity premium implies that the equity price must fall dramatically on impact, about 4.5 percent.[42] Thus, the model also satisfies the second criterion described above—that an exogenous increase in volatility causes stock prices to fall—without the need for an IES > 1, consistent with the discussion above even though the model differs from BY in many respects.

The exogenous shock to volatility in Figure 9 is related to the literature on "uncertainty shocks", such as Bloom (2009), who shows that recessions are typically associated with greater stock market uncertainty. One interpretation of this correlation is that exogenous increases in uncertainty lead to recessions, as in Figure 9 (where consumption and output about 0.25 percent in response to the shock), but it is also possible that the causality runs both ways, so that recessions lead to higher stock market volatility and uncertainty (e.g., Ludvigson, Ma, and Ng, 2021).

In fact, both of these channels can be seen in the model developed above. In addition to the impulse responses in Figure 9 to an exogenous uncertainty shock, the model here implies that recessions cause stock market uncertainty to increase *endogenously*.. The intuition for this is essentially the same as for endogenous conditional heteroskedasticity, above. When the economy is weak, consumption is low and the household's stochastic discount factor becomes more sensitive to subsequent shocks. This drives up the equity premium, as shown in Figure 2, but it also increases the uncertainty about stock prices, as shown in Figure 10, which plots the nonlinear impulse response function of the one-step-ahead standard deviation of the log equity price, $\mathrm{sd}_t\, p_{t+1}^e$ to a one-standard-deviation (0.7 percent) technology shock, computed the same way as in previous figures. In response to a positive technology shock, uncertainty about the equity price in the

[42] In order to generate an equity premium of 60 bp in the first period, stock prices must fall by about 0.6 percent below their second-period value. In order to generate an equity premium in each subsequent period, equity prices must continue to rise. This requires a large initial fall in the equity price so that in each subsequent period equity prices can rise in line with the implied equity premium.

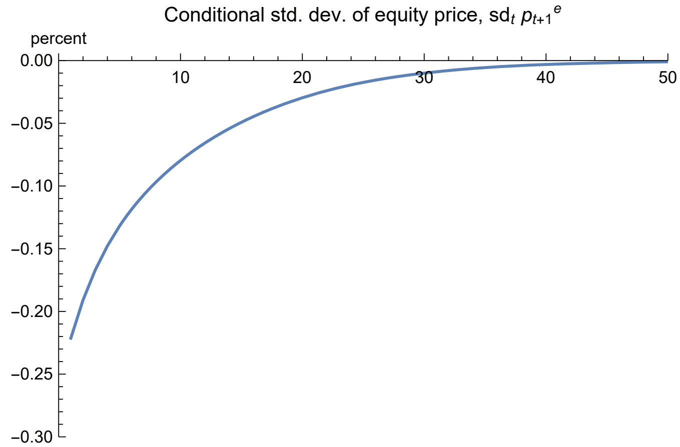

Figure 10. Nonlinear impulse response function for the one-period-ahead standard deviation of the log equity price, $\mathrm{sd}_t\, p^e_{t+1}$, to a one-standard-deviation (0.7 percent) technology shock in the baseline model with state variables initialized to their steady-state values. See text for details.

next period decreases about 0.2 percent—i.e., from about 2.5 percent in Figure 2 to less than 2.3 percent immediately after the shock—a substantial reduction.

4.3 The Financial Accelerator

Traditional models of the financial accelerator (e.g., Bernanke, Gertler, and Gilchrist, 1999; Kiyotaki and Moore, 1997; Gertler and Kiyotaki, 2011) allow for the possiblity of default, but ignore deviations from risk neutrality. The fact that borrowers might default introduces a wedge between borrowers and lenders that can act as an amplification and propagation mechanism for shocks: for example, in Kiyotaki and Moore (1997), a negative technology shock reduces the value of capital, which reduces firms' collateral; with less collateral, firms must scale back production and output falls by more than the effect of the technology shock alone. In Bernanke, Gertler, and Gilchrist (1999, henceforth BGG), a weaker economy implies a higher probability of default, which raises costly state verification costs for financial intermediaries, which in turn leads to a greater spread between private borrowing rates and the risk-free rate.

The traditional financial accelerator mechanism captures many important features of a credit crunch and a financial crisis. At the same time, these models are essentially risk-neutral and abstract from risk premia—that is, in BGG and Kiyotaki and Moore (1997), the spread between private borrowing rates and the risk-free rate is just the risk-neutral expected loss from default. Yet an important part of the transmission mechanism in the 2007–08 financial crisis was the fall in the value of collateral beyond even the risk-neutral probability of default: for example, risk and liquidity premia rose dramatically even on securities that had little or no connection to subprime real estate lending (Gorton and Metrick, 2012), and the prices of many mortgage-backed securities and credit default swaps fell by much more than can be explained by any reasonable assumption for mortgage default and recovery rates (Stanton and Wallace, 2011). The dramatic increase in risk premia during the financial crisis caused huge drops in the value of collateral and historic increases in credit spreads. For the same reasons as in traditional financial accelerator models, we would expect these repercussions from rising risk premia to be an important part of the transmission mechanism from financial markets to the real economy.

Recently, some authors have begun to incorporate deviations from risk neutrality into financial accelerator models—see, e.g., He and Krishnamurthy (2013), Brunnermeier and Sannikov (2014)—but those models are idiosyncratic. In contrast, the macroeconomic model I've developed here is canonical and would allow researchers to study the effects of risk premia on collateral values, credit spreads, and the economy within the well-understood and ubiquitous New Keynesian DSGE framework. Extending the model to include a financial intermediation sector is beyond the scope of the present paper, but would be a worthy topic for future research.

5. Conclusions

This paper shows that a simple, textbook New Keyneisan model with Epstein-Zin preferences is consistent with a wide variety of asset pricing facts, such as the equity premium, real and nominal term premium, and credit spread. The basic New Keynesian model produces the correct correlations between the macroeconomy, real and nominal interest rates, and dividends, while Epstein-Zin preferences allow the model to match the overall size and variability of the risk premia on these assets.

For simplicity, I have kept the model as parsimonious as possible and used Epstein-Zin preferences with a high degree of risk aversion to match the size of risk premia in the data. This degree of risk aversion can be lowered dramatically by augmenting the model to include additional sources of risk. For example, Barillas, Hansen, and Sargent (2009) show there is a formal mathematical equivalence between Epstein-Zin preferences with a high degree of risk aversion and those same preferences with a lower degree of risk aversion but a moderate level of uncertainty about the structure and parameters of the model. Similarly, Schmidt (2015), following Constantinides and Duffie (1996), shows that Epstein-Zin preferences with a high degree of risk aversion in a representative agent model are formally equivalent to those same preferences with a lower degree of risk aversion and heterogeneous agents who face uninsurable idiosyncratic risks. One can similarly reduce the required risk aversion in the model by adding long-run risks (e.g., Bansal and Yaron, 2004), rare disasters (e.g., Rietz, 1988; Barro, 2006), or parameter uncertainty (e.g., Weitzman, 2007). All of these approaches address the fact that the quantity of risk faced by a representative household in a textbook New Keynesian model with i.i.d. normally-distributed shocks is not very large. The point of the present paper is not to incorporate all of these additional features, but rather to serve as a "proof of concept" that the asset pricing data can be matched within a standard New Keynesian framework.

The simple, structural model I develop provides a unified and intuitive framework for thinking about asset prices and asset pricing puzzles. Rather than studying each puzzle in isolation, the model here provides a reasonable description of the behavior of all of the major asset classes. In addition, structural models have the advantage of being more robust to structural breaks and novel policy interventions, such as those observed during the recent global financial crisis and European sovereign debt crisis. The model developed here can potentially provide insight in these situations, when more traditional, reduced-form models are largely uninformative.

Finally, by showing how a standard macroeconomic model can be made consistent with the behavior of risk premia in financial markets, the present paper opens the door to studying the feedback between those risk premia and the economy within the standard macreconomic modeling framework. As evidenced by the recent financial crises, this feedback from asset prices to the economy and back again can be very important. In the simple, stylized model of the present paper, asset prices have no feedback effects on the real economy, for simplicity, but it would be very interesting to combine the asset-pricing framework of the present paper with a macroeconomic model that includes a financial accelerator along the lines of Bernanke, Gertler, and Gilchrist (1999), Kiyotaki and Moore (1997), Gertler and Kiyotaki (2011), and others. In general, these models abstract from risk aversion and risk premia and focus instead on the effect of agency problems and collateral constraints on lending and investment. In a combined framework, shocks that cause the economy to deteriorate would lead to an increase in risk premia and a concomitant fall in asset prices, further amplifying the collateral constraint on firms and financial intermediaries. This channel appears to have been an important amplification and propagation mechanism in the recent crises.

Appendix A: Model Equations

We can write the equations of the macroeconomic model in Section 2 in recursive form as follows. (Equations for equity and debt are essentially the same as in Section 3 and are not reproduced here.)

Value function:

$$V_t \;=\; (1-\beta)\left(\log C_t - \eta\,\frac{L_t^{1+\chi}}{(1+\chi)}\right) - \beta\alpha^{-1}\log \mathrm{Vexp}_t, \tag{A1}$$

$$\mathrm{Vexp}_t \;=\; E_t \exp(-\alpha V_{t+1}). \tag{A2}$$

Risk-free real rate and Euler equations:

$$e^{-r_t} \;=\; \beta E_t (C_{t+1}/C_t)^{-1}\big(\exp(-\alpha V_{t+1})/\mathrm{Vexp}_t\big), \tag{A3}$$

$$C_t^{-1} \;=\; \beta E_t e^{i_t-\pi_{t+1}} C_{t+1}^{-1}\big(\exp(-\alpha V_{t+1})/\mathrm{Vexp}_t\big). \tag{A4}$$

Optimal price setting by firms:

$$\left(\frac{p_t^*}{P_t}\right)^{1+\lambda(1-\theta)/((\lambda-1)\theta)} \;=\; \lambda\,\frac{z_t^n}{z_t^d}\,, \tag{A5}$$

$$z_t^n \;=\; \mu_t Y_t + \beta\xi E_t (C_{t+1}/C_t)^{-1}\big(\exp(-\alpha V_{t+1})/\mathrm{Vexp}_t\big)(e^{\pi_{t+1}-\bar\pi})^{\lambda/((\lambda-1)\theta)} z_{t+1}^n, \tag{A6}$$

$$z_t^d \;=\; Y_t + \beta\xi E_t (C_{t+1}/C_t)^{-1}\big(\exp(-\alpha V_{t+1})/\mathrm{Vexp}_t\big)(e^{\pi_{t+1}-\bar\pi})^{1/(\lambda-1)} z_{t+1}^d, \tag{A7}$$

$$(e^{\pi_t-\bar\pi})^{1/(1-\lambda)} \;=\; (1-\xi)\left(\frac{p_t^*}{P_t}\right)^{1/(1-\lambda)}(e^{\pi_t-\bar\pi})^{1/(1-\lambda)} + \xi. \tag{A8}$$

Marginal cost and real wage:

$$\mu_t \;=\; \frac{w_t}{P_t}\,\frac{Y_t^{(1-\theta)/\theta}}{\theta A_t^{1/\theta} K^{(1-\theta)/\theta}}\,, \tag{A9}$$

$$\eta L_t^{\chi}/C_t^{-1} \;=\; \frac{w_t}{P_t}. \tag{A10}$$

Production and resource constraint:

$$Y_t \;=\; A_t K^{1-\theta} L_t^{\theta}/\Delta_t, \tag{A11}$$

$$\Delta_t^{1/\theta} \;=\; (1-\xi)\left(\frac{p_t^*}{P_t}\right)^{-\lambda/((\lambda-1)\theta)} + \xi(e^{\pi_t-\bar\pi})^{\lambda/((\lambda-1)\theta)}\Delta_{t-1}^{1/\theta}, \tag{A12}$$

$$Y_t \;=\; C_t. \tag{A13}$$

Monetary policy rule:

$$i_t \;=\; \log(1/\beta) + \pi_t + \phi_\pi(\pi_t - \bar\pi) + \frac{\phi_y}{4}\log(Y_t/\bar Y_t), \tag{A14}$$

$$\log \bar Y_t \;=\; \rho_{\bar y}\log \bar Y_{t-1} + (1-\rho_{\bar y})\log Y_t. \tag{A15}$$

Technology shock:

$$\log A_t \;=\; \log A_{t-1} + \varepsilon_t^A. \tag{A16}$$

Equations (A1)–(A2) break the generalized value function into two equations to correspond to the syntax of Perturbation AIM and other rational expectations equation solvers, which typically require the model to be written as a system of equations in a form similar to $E_t F(X_{t-1}, X_t, X_{t+1}; \varepsilon_t) = 0$.

Equations (A5)–(A7) represent monopolistic firms' optimal price-setting equations. The exponent on (p_t^*/P_t) in (A5) follows from substituting out $y_{t+j}(f)$ in equation (14) in the main text, and is due to the presence of firm-specific capital stocks. Equations (A6)–(A7) are recursive versions of the infinite sums in the numerator and denominator of (14).

The other equations above follow in a straightforward manner from the equations in the main text.

Although capital stocks in the model above are fixed, the model nevertheless has a balanced growth path along which all variables are either constant or grow at constant rates if technology A_t itself grows at a constant rate. Along the balanced growth path, each of the variables Y_t, C_t, w_t, $\bar{Y}_t$, z_t^n, and z_t^d grow at the same rate as A_t. If we divide each of these variables through by A_t, the ratios have a nonstochastic steady state. Moreover, after a shock to A_t, these ratios converge back to their pre-shock levels. Thus, the nonstochastic steady state of these ratios constitutes a stable point around which we can approximate the model.

I thus transform the model by dividing each of the above variables by the level of technology A_t, and transform the value function V_t by defining $\tilde{V}_t \equiv V_t - \log A_t$. The transformed model then has a nonstochastic steady state around which I can compute an nth-order approximate solution as described in the text. These solutions are highly accurate in a neighborhood of the steady state, and become increasingly accurate over larger regions of the state space as the order of approximation n becomes large (see Swanson, Anderson, and Levin, 2006, for details and discussion).

Appendix B: Impulse Responses to a Negative Technology Shock

The nonlinear impulse response functions graphed in Figures 1 through 5 can be asymmetric for positive and negative shocks, because they are nonlinear. In practice, the New Keynesian model presented in the main text does not produce impulse response functions that are very asymmetric. This can be seen, for example, in Figure/ B1, which reproduces Figure 1 from the main text for the case of a negative one-standard-deviation ($-.007$) shock to technology A_t. The nonlinear impulse response functions in Figure B1 are computed in exactly the same was as in Figures 1 through 6, except that the shock has the opposite sign. Overall, the responses in Figure B1 are close to being symmetric counterparts to Figure 1.

Figure B2 presents nonlinear impulse response functions for the equity price p_t^e, equity premium ψ_t^e, real long-term bond price $p_t^{(40)}$, and term premium $\psi_t^{(40)}$ on the real long-term bond to the negative one-standard-deviation technology shock. The impulse responses are again close to being the symmetric counterparts to the nonlinear impulse responses functions in Figures 2 and 3, although the asset price and risk premium responses are slightly larger in magnitude for the negative shock than they are for the positive shock. For example, the equity premium increases by about 70 bp after the negative technology shock here, while it fell by about 62 bp after the positive shock in Figure 2. Similarly, the equity price falls by about 2.75 percent after the negative technology shock here, but rose by about 2.5 percent after the positive shock in Figure 2.

Figure B3 repeats the analysis for the nominal long-term bond price $p_t^{\$(40)}$, the term premium $\psi_t^{\$(40)}$ on the nominal long-term bond, defaultable bond price p_t^d, and credit spread $i_t^d - i_t^c$. As with Figure B2, the responses here are essentially symmetric to their counterparts in Figures 4 and 5, while also being somewhat larger in magnitude.

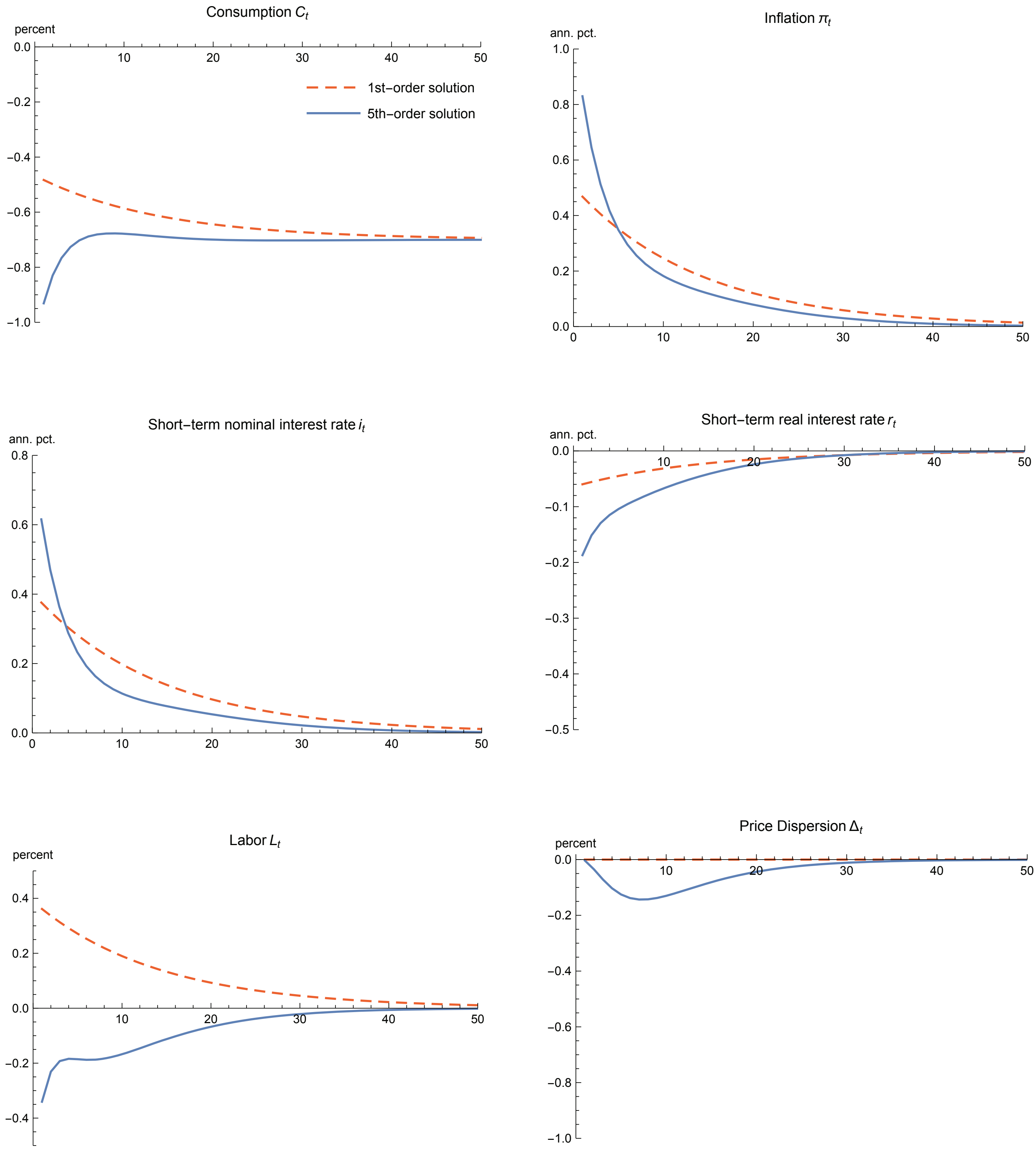

Figure B1. First-order (dashed red lines) and fifth-order (solid blue lines) impulse response functions for consumption C_t, inflation π_t, short-term nominal interest rate i_t, short-term real interest rate r_t, labor L_t, and price dispersion Δ_t to a one-standard-deviation *negative* (-0.7 percent) technology shock in the model. See Figure 1 for comparison and text for details.

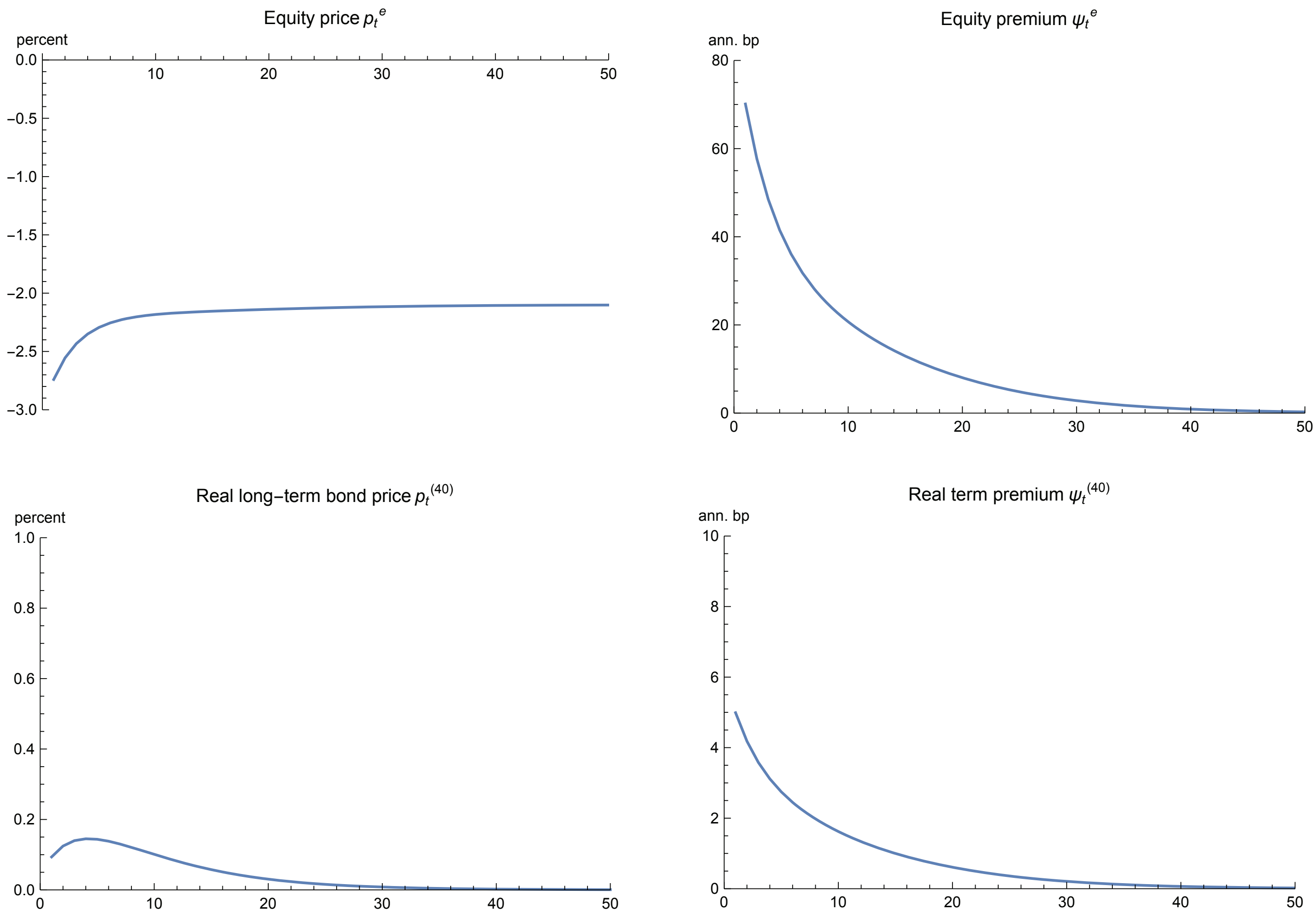

Figure B2. Nonlinear impulse response functions for the equity price p_t^e, equity premium ψ_t^e, real long-term bond price $p_t^{(40)}$ and real term premium $\psi_t^{(40)}$ to a one-standard-deviation *negative* (-0.7 percent) technology shock in the model, with state variables initialized to their nonstochastic steady state values. See Figures 2–3 for comparison and text for details.

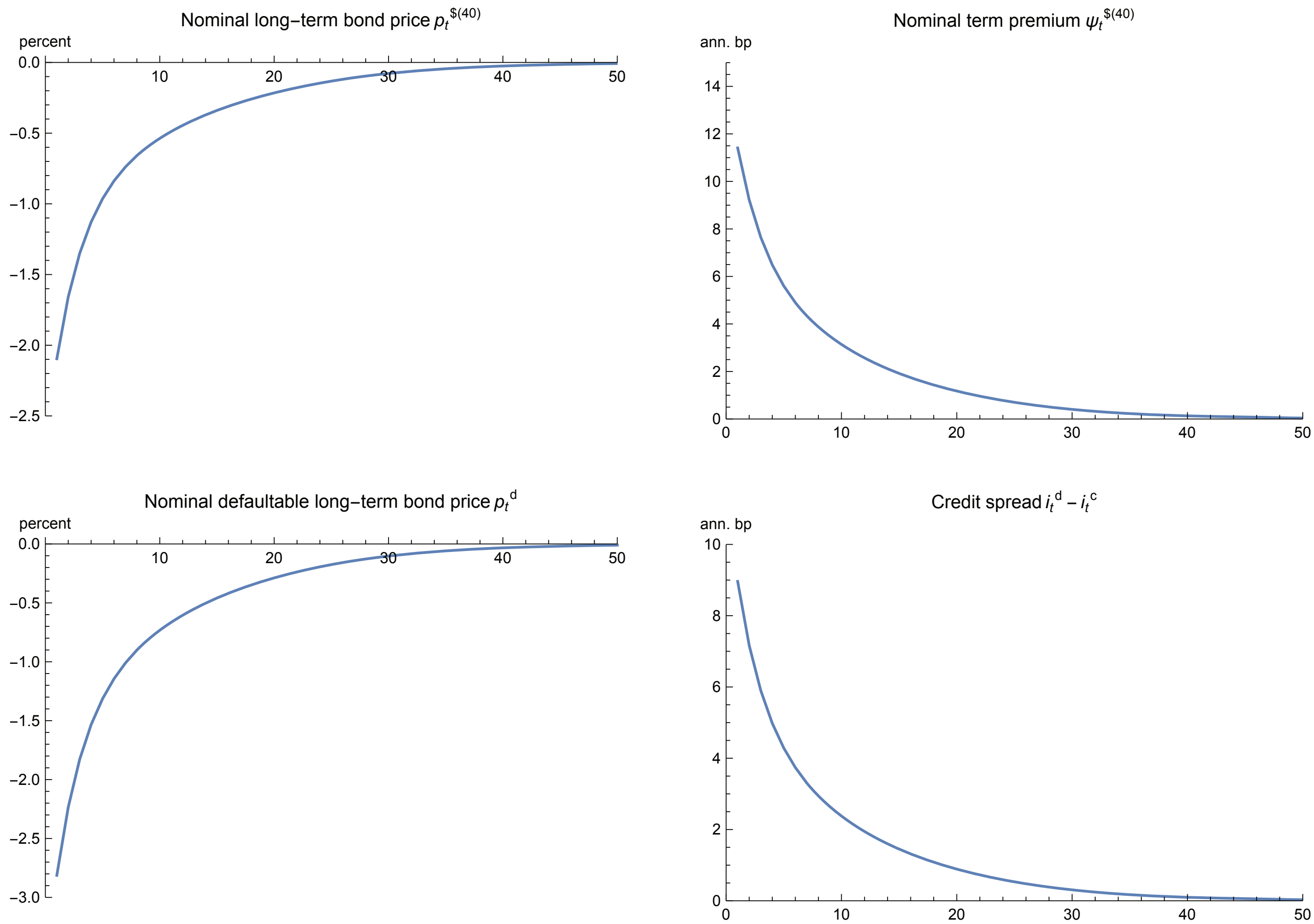

Figure B3. Nonlinear impulse response functions for the nominal long-term bond price $p_t^{\$(40)}$, nominal bond term premium $\psi_t^{\$(40)}$, defaultable long-term bond price p_t^d, and credit spread $i_t^d - i_t^c$ to a one-standard-deviation *negative* (-0.7 percent) technology shock in the model, with state variables initialized to their nonstochastic steady state values. See Figures 4–5 for comparison and text for details.

References

Abel, Andrew (1999). "Risk Premia and Term Premia in General Equilibrium," *Journal of Monetary Economics* 43, 3–33.

Adrian, Tobias, Erkko Etula, and Tyler Muir (2014). "Financial Intermediaries and the Cross-Section of Asset Returns," *Journal of Finance* 69, 2557–2596.

Altig, David, Lawrence Christiano, Martin Eichenbaum, and Jesper Lindé (2011). "Firm-Specific Capital, Nominal Rigidities, and the Business Cycle," *Review of Economic Dynamics* 14, 225–247.

Altonji, Joseph (1986). "Intertemporal Substitution in Labor Supply: Evidence from Micro Data," *Journal of Political Economy* 94, S176–S215.

Andreasen, Martin (2012). "An Estimated DSGE Model: Explaining Variation in Nominal Term Premia, Real Term Premia, and Inflation Risk Premia," *European Economic Review* 56, 1656–1674.

Ang, Andrew, and Maxim Ulrich (2012). "Nominal Bonds, Real Bonds, and Equity," unpublished manuscript, Columbia Business School.

Aruoba, S. Boragan, Jesús Fernández-Villaverde, and Juan Rubio-Ramírez (2006). "Comparing Solution Methods for Dynamic Equilibrium Economies," *Journal of Economic Dynamics and Control* 30, 2477–2508.

Atanasov, Victoria, Stig Moller, and Richard Priestley (2020). "Consumption Fluctuations and Expected Returns," *Journal of Finance* 75, 1677–1713.

Backus, David, Allan Gregory, and Stanley Zin (1989). "Risk Premiums in the Term Structure," *Journal of Monetary Economics* 24, 371–399.

Baele, Lieven, Geert Bekaert, and Koen Inghelbrecht (2010). "The Determinants of Stock and Bond Comovements," *Review of Financial Studies* 23, 2374–2428.

Bansal, Ravi, and Amir Yaron (2004). "Risks for the Long Run: A Potential Resolution of Asset-Pricing Puzzles," *Journal of Finance* 59, 1481–1509.

Barillas, Francisco, Lars Hansen, and Thomas Sargent (2009). "Doubts or Variability?" *Journal of Economic Theory* 144, 2388–2418.

Barro, Robert (2006). "Rare Disasters and Asset Markets in the Twentieth Century," *Quarterly Journal of Economics* 121, 823–866.

Bekaert, Geert, Seonghoon Cho, and Antonio Moreno (2010). "New Keynesian Macroeconomics and the Term Structure," *Journal of Money, Credit and Banking* 42, 33–62.

Bekaert, Geert, Eric Engstrom, and Steven Grenadier (2010). "Stock and Bond Returns with Moody Investors," *Journal of Empirical Finance* 17, 867–894.

Bernanke, Ben, Mark Gertler, and Simon Gilchrist (1999). "The Financial Accelerator in a Quantitative Business Cycle Framework," *Handbook of Macroeconomics* 1, 1341–1393.

Bhamra, Harjoat, Lars-Alexander Kuehn, and Ilya Strebulaev (2010). "The Levered Equity Risk Premium and Credit Spreads: A Unified Framework," *Review of Financial Studies* 23, 645–703.

Bloom, Nicholas (2009). "The Impact of Uncertainty Shocks," *Econometrica* 77, 623–685.

Boldrin, Michele, Lawrence Christiano, and Jonas Fisher (2001). "Habit Persistence, Asset Returns, and the Business Cycle," *American Economic Review* 91, 149–166.

Brunnermeier, Markus, and Yuliy Sannikov (2014). "A Macroeconomic Model with a Financial Sector," *American Economic Review* 104, 379–421.

Calvo, Guillermo (1983). "Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics* 12, 383–398.

Campanale, Claudio, Rui Castro, and Gian Luca Clementi (2010). "Asset Pricing in a Production Economy with Chew-Dekel Preferences," *Review of Economic Dynamics* 13, 379–402.

Campbell, John (1999). "Asset Prices, Consumption, and the Business Cycle," *Handbook of Macroeconomics* 1, ed. John Taylor and Michael Woodford, Elsevier, 1231–1303.

Campbell, John, and John Cochrane (1999). "By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior," *Journal of Political Economy* 107, 205–251.

Campbell, John, Carolin Pflueger, and Luis Viceira (2020). "Macroeconomic Drivers of Bond and Equity Risks," *Journal of Political Economy* 128, 3148–3185.

Campbell, John, Adi Sundaram, and Luis Viceira (2017). "Inflation Bets or Deflation Hedges? The Changing Risks of Nominal Bonds," *Critical Finance Review* 6, 263–301.

Chen, Hui (2010). "Macroeconomic Conditions and the Puzzles of Credit Spreads and Capital Structure," *Journal of Finance* 65, 2171–2212.

Chen, Long, Pierre Collin-Dufresne, and Robert Goldstein (2009). "On the Relation Between the Credit Spread Puzzle and the Equity Premium Puzzle," *Review of Financial Studies* 22, 3367–3409.

Christiano, Lawrence, Martin Eichenbaum, and Charles Evans (2005). "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy* 113, 1–45.

Clarida, Richard, Jordi Galí, and Mark Gertler (2000). "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory," *Quarterly Journal of Economics* 115, 147–180.

Cochrane, John (2008). "Financial Markets and the Real Economy," *Handbook of the Equity Risk Premium,* 237–326.

Cochrane, John, and Monika Piazzesi (2005). "Bond Risk Premia," *American Economic Review* 95, 138–160.

Constantinides, George, and Darrell Duffie (1996). "Asset Pricing with Heterogeneous Consumers," *Journal of Political Economy* 104, 219–240.

Cooper, Ilan, and Richard Priestley (2008). "Time-Varying Risk Premiums and the Output Gap," *Review of Financial Studies* 22, 2801–2833.

Corhay, Alexandre, Thilo Kind, Howard Kung, and Gonazlo Morales (2019). "Discount Rates, Debt Maturity, and the Fiscal Theory," unpublished manuscript, London Business School.

Dai, Qiang, and Kenneth Singleton (2003). "Fixed-Income Pricing," *Handbook of the Economics of Finance* 1B, 1207–1246.

Del Negro, Marco, Marc Giannoni, and Frank Schorfheide (2015). "Inflation in the Great Recession and New Keynesian Models," *American Economic Journal: Macroeconomics* 7, 168–196.

Den Haan, Wouter (1995). "The Term Structure of Interest Rates in Real and Monetary Economies," *Journal of Economic Dynamics and Control* 19, 909–940.

Dew-Becker, Ian (2014). "A Model of Time-Varying Risk Premia with Habits and Production," unpublished manuscript, Kellogg School of Business, Northwestern University.

Donaldson, John, Thore Johnsen, and Rajnish Mehra (1990). "On the Term Structure of Interest Rates," *Journal of Economic Dynamics and Control* 14, 571–596.

Duffie, Darrell, and David Lando (2001). "Term Structures of Credit Spreads with Incomplete Accounting Information," *Econometrica* 69, 633–664.

Epstein, Larry, and Stanley Zin (1989). "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework," *Econometrica* 57, 937–969.

Eraker, Bjorn (2008). "Affine General Equilibrium Models," *Management Science* 54, 2068–2080.

European Central Bank (2012). "Technical Features of Outright Monetary Transactions," press release, 6 September, http://www.ecp.europa.eu/press/pr/date/2012/html/pr120906_1.en.html.

Fama, Eugene, and Kenneth French (1989). "Business Conditions and Expected Returns on Stocks and Bonds," *Journal of Financial Economics* 25, 23–49.

Fama, Eugene, and Kenneth French (2002). "The Equity Premium," *Journal of Finance* 57, 637–659.

Fama, Eugene, and Kenneth French (2015). "A Five-Factor Asset Pricing Model," *Journal of Financial Economics* 116, 1–22.

Fleckenstein, Matthias, Francis Longstaff, and Hanno Lustig (2014). "The TIPS-Treasury Bond Puzzle," *Journal of Finance* 69, 2151–2197.

Galí, Jordi (2008). *Monetary Policy, Inflation, and the Business Cycle,* Princeton University Press.

Gertler, Mark, and Nobuhiro Kiyotaki (2011). "Financial Intermediation and Credit Policy in Business Cycle Analysis," *Handbook of Monetary Economics* 3A, 547–599.

Giesecke, Kay, Francis Longstaff, Stephen Schaefer, and Ilya Strebulaev (2011). "Corporate Bond Default Risk: A 150-Year Perspective," *Journal of Financial Economics* 102, 233–250.

Gorton, Gary, and Andrew Metrick (2012). "Getting Up to Speed on the Financial Crisis: A One-Weekend-Reader's Guide," *Journal of Economic Literature* 50, 128–150.

Gourio, Francois (2012). "Disaster Risk and Business Cycles," *American Economic Review* 102, 2734–2766.

Greenwood, Jeremy, Zvi Hercowitz, and Gregory Huffman (1998). "Investment, Capacity Utilization, and the Real Business Cycle," *American Economic Review* 78, 402–417.

Gürkaynak, Refet, Brian Sack, and Jonathan Wright (2007). "The U.S. Treasury Yield Curve: 1961 to the Present," *Journal of Monetary Economics* 54, 2291–2304.

Gürkaynak, Refet, Brian Sack, and Jonathan Wright (2010). "The TIPS Yield Curve and Inflation Compensation," *American Economic Journal: Macroeconomics* 2, 70–92.

Guvenen, Fatih (2009). "A Parsimonious Macroeconomic Model for Asset Pricing," *Econometrica* 77, 1711–1750.

Hall, Robert (1988). "Intertemporal Substitution in Consumption," *Journal of Political Economy* 96, 339–357.

Hansen, Lars P. and Thomas Sargent (2001). "Robust Control and Model Uncertainty," *American Economic Review* 91, 60–66.

He, Zhiguo, and Arvind Krishnamurthy. "Intermediary Asset Pricing," *American Economic Review* 103 (2013), 732–770.

Judd, Kenneth (1988). *Numerical Methods in Economics,* MIT Press.

Kim, Don, and Jonathan Wright (2005). "An Arbitrage-Free Three-Factor Term Structure Model and the Recent Behavior of Long-Term Yields and Distant-Horizon Forward Rates," *Federal Reserve Board Finance and Economics Discussion Series* 2005–33.

King, Robert, Charles Plosser, and Sergio Rebelo (1988). "Production, Growth, and Business Cycles: I. The Basic Neoclassical Model," *Journal of Monetary Economics* 21, 195–232.

King, Robert, Charles Plosser, and Sergio Rebelo (2002). "Production, Growth, and Business Cycles: Technical Appendix," *Computational Economics* 20, 87–116.

King, Robert, and Sergio Rebelo (1999). "Resuscitating Real Business Cycles," in *Handbook of Macroeconomics* 1, ed. John Taylor and Michael Woodford, Elsevier, 927–1007.

KIYOTAKI, NOBUHIRO, AND JOHN MOORE (1997). "Credit Cycles," *Journal of Political Economy* 105, 211–248.

KOIJEN, RALPH, HANNO LUSTIG, AND STIJN VAN NIEUWERBURGH (2013). "The Cross-Section and Time Series of Stock and Bond Returns," *NBER Working Paper* 15688.

KYDLAND, FINN, AND EDWARD PRESCOTT (1982). "Time to Build and Aggregate Fluctuations," *Econometrica* 50, 1345–1370.

LANE, PHILLIP (2012). "The European Sovereign Debt Crisis," *Journal of Economic Perspectives* 26, 49–68.

LELAND, HAYNE (1994). "Corporate Debt Value, Bond Covenants, and Optimal Capital Structure," *Journal of Finance* 49, 1213–1252.

LELAND, HAYNE (1998). "Agency Costs, Risk Management, and Capital Structure," *Journal of Finance* 53, 1213–1243.

LETTAU, MARTIN, AND SYDNEY LUDVIGSON (2010). "Measuring and Modeling Variation in the Risk-Return Trade-off," *Handbook of Financial Econometrics* 1, 617–690.

LETTAU, MARTIN, AND HARALD UHLIG (2000). "Can Habit Formation Be Reconciled with Business Cycle Facts?" *Review of Economic Dynamics* 3, 79–99.

LETTAU, MARTIN, AND JESSICA WACHTER (2011). "The Term Structure of Equity and Interest Rates," *Journal of Financial Economics* 101, 90–113.

LUDVIGSON, SYDNEY, SAI MA, AND SERENA NG (2021). "Uncertainty and Business Cycles: Exogenous Impulse or Endogenous Response?" *American Economic Journal: Macroeconomics,* forthcoming.

MACURDY, THOMAS (1981). "An Empirical Model of Labor Supply in a Life-Cycle Setting," *Journal of Political Economy* 89, 1059–1085.

MALLOY, CHRISTOPHER, TOBIAS MOSKOWITZ, AND ANNETTE VISSING-JORGENSEN (2009). "Long-Run Stockholder Consumption, Risk and Asset Returns," *Journal of Finance* 44, 2427–2479.

MANKIW, N. GREGORY, AND STEPHEN ZELDES (1991). "The Consumption of Stockholders and Non-stockholders," *Journal of Financial Economics* 29, 97–112.

MEHRA, RAJNISH, AND EDWARD PRESCOTT (1985). "The Equity Premium: A Puzzle," *Journal of Monetary Economics* 15, 145–161.

MISHKIN, FREDERIC (2011). "Over the Cliff: From the Subprime to the Global Financial Crisis," *Journal of Economic Perspectives* 25, 49–70.

PARKER, JONATHAN (2001). "The Consumption Risk of the Stock Market," *Brookings Papers on Economic Activity* 2, 279–333.

PFLUEGER, CAROLIN, AND GIANLUCA RINALDI (2021). "Why Does the Fed Move Markets So Much? A Model of Monetary Policy and Time-Varying Risk Aversion," unpublished manuscript, Harris School of Public Policy, University of Chicago.

PIAZZESI, MONIKA, AND MARTIN SCHNEIDER (2006). "Equilibrium Yield Curves," *NBER Macroeconomics Annual* 21, 389–442.

PIAZZESI, MONIKA, AND ERIC SWANSON. "Futures Prices as Risk-Adjusted Forecasts of Monetary Policy," *Journal of Monetary Economics* 55 (2008), 677–691.

RIETZ, THOMAS (1988). "The Equity Risk Premium: A Solution," *Journal of Monetary Economics* 22, 117–131.

ROLL, RICHARD (1977). "A Critique of the Asset Pricing Theory's Tests Part I: On Past and Potential Testability of the Theory," *Journal of Financial Economics* 4, 129–176.

Rudebusch, Glenn (2002). "Term Structure Evidence on Interest Rate Smoothing and Monetary Policy Inertia," *Journal of Monetary Economics* 49, 1161–1187.

Rudebusch, Glenn, Brian Sack, and Eric Swanson (2007). "Macroeconomic Implications of Changes in the Term Premium," *Federal Reserve Bank of St. Louis Economic Review* 89, 241–269.

Rudebusch, Glenn, and Eric Swanson (2008). "Examining the Bond Premium Puzzle with a DSGE Model," *Journal of Monetary Economics* 55, 111–126.

Rudebusch, Glenn, and Eric Swanson (2012). "The Bond Premium in a DSGE Model with Long-Run Real and Nominal Risks," *American Economic Journal: Macroeconomics* 4, 105–143.

Rudebusch, Glenn, and Tao Wu (2007). "Accounting for a Shift in Term-Structure Behavior with No-Arbitrage Macro-Finance Models," *Journal of Money, Credit and Banking* 39, 395–422.

Schmidt, Lawrence (2015). "Climbing and Falling Off the Ladder: Asset Pricing Implications of Labor Market Event Risk," unpublished manuscript, University of Chicago.

Smets, Frank, and Rafael Wouters (2007). "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *American Economic Review* 97, 586–606.

Standard & Poor's (2011). *2011 Annual Global Corporate Default Study and Rating Transitions,* New York.

Stanton, Richard, and Nancy Wallace (2011). "The Bear's Lair: Index Credit Default Swaps and the Subprime Mortgage Crisis," *Review of Financial Studies* 24, 3250–3280.

Swanson, Eric (2018). "Risk Aversion, Risk Premia, and the Labor Margin with Generalized Recursive Preferences," *Review of Economic Dynamics* 28, 290–321.

Swanson, Eric, Gary Anderson, and Andrew Levin (2006). "Higher-Order Perturbation Solutions to Dynamic, Discrete-Time Rational Expectations Models," *Federal Reserve Bank of San Francisco Working Paper* 2006–01.

Tallarini, Thomas (2001). "Risk-Sensitive Business Cycles," *Journal of Monetary Economics* 45, 507–532.

Taylor, John (1993). "Discretion versus Policy Rules in Practice," *Carnegie-Rochester Conference Series on Public Policy* 39, 195–214.

Taylor, John (1999). "A Historical Analysis of Monetary Policy Rules," in *Monetary Policy Rules,* ed. John Taylor (MIT Press: Cambridge MA), 319–341.

Van Binsbergen, Jules, Jesús Fernández-Villaverde, Ralph Koijen, and Juan Rubio-Ramírez (2012). "The Term Structure of Interest Rates in a DSGE Model with Recursive Preferences," *Journal of Monetary Economics* 59, 634–648.

Vissing-Jorgensen, Annette (2002). "Limited Asset Market Participation and the Elasticity of Substitution," *Journal of Political Economy* 110, 825–853.

The Wall Street Journal (2012). "Secret Weapon: Europe's Loan Plan," Tom Lauricella and Matt Phillips, January 23.

Weil, Phillipe (1989). "The Equity Premium Puzzle and the Risk-Free Rate Puzzle," *Journal of Monetary Economics* 24, 401–421.

Weitzman, Martin (2007). "Subjective Expectations and Asset-Return Puzzles," *American Economic Review* 97, 1102–1130.

Woodford, Michael (2003). *Interest and Prices,* Princeton University Press.