NAIRU Uncertainty and Nonlinear Policy Rules

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Core inflation in the United States declined from the end of 1995 through the end of 1999 and has remained modest thereafter despite persistent above-trend growth and a decline in the unemployment rate to a level well below earlier estimates of the non-accelerating-inflation rate of unemployment (NAIRU).[1] These developments have had at least two implications for the conduct of monetary policy. First, policymakers (at least those that have continued to rely on the NAIRU framework) have had to update their estimates of the NAIRU as they have decided how much to adjust the federal funds rate in response to realizations in inflation and unemployment. Second, policymakers have had to adjust the conduct of monetary policy to take into account the heightened uncertainty about the NAIRU.

Although a standard result in the literature on monetary policy under uncertainty is that of certainty-equivalence, a recent strand of this literature has focused on signal extraction in the estimation stage of the policymakers' problem. This part of the literature suggests that policymakers should attenuate their response to changes in the observed unemployment rate when they are more uncertain about the NAIRU and, at the same time, respond more aggressively to movements in the inflation rate (Swanson, 2000a).

In addition, Meyer (1999) has suggested that episodes of heightened uncertainty about the NAIRU may also warrant a *nonlinear* policy response to changes in the unemployment rate. Specifically, he has suggested that in such circumstances, policymakers: (i) initially attenuate their response to changes in the unemployment rate, but (ii) return to a more aggressive policy response when the unemployment rate falls far enough that policymakers regain confidence that it lies below the NAIRU. This paper is an attempt to formalize such a nonlinear policy rule and test its performance when there is heightened uncertainty about the NAIRU.

I. Optimal Nonlinear Policy

To illustrate the basic point of the paper without introducing unnecessary complications, we use a simple backward-looking model (in simulations we also consider models with richer dynamics and rational expectations). This model consists of an "*IS*" type equation relating the lagged real interest rate r_{t-1} to unemployment u_t, and a short-run Phillips curve that determines inflation π_t:

$$(1)\quad (u_t - u^*) = \theta(u_{t-1} - u^*) + \alpha(r_{t-1} - r^*) + \varepsilon_t$$

$$(2)\quad \pi_t = \pi_{t-1} - \beta(u_{t-1} - u^*) + \nu_t$$

where r^*, θ, α, and β are known parameters, ε_t and ν_t are stochastic disturbances, and u^* denotes the NAIRU. Policymakers never observe u^* directly but must infer it from observations of u and π.

We maintain the standard assumption that policymakers' preferences are quadratic over inflation and unemployment gaps. Optimal policy in this standard linear-quadratic model is then given by

$$(3)\quad r_t = r^* + a(\pi_t - \pi^*) - b(u_t - E_t u^*)$$

where we assume that policymakers control the real interest rate r_t. This policy displays the usual property of certainty-equivalence: interest

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[1] The single most important contributor to the exceptional macroeconomic performance in recent years appears to be a dramatic increase in the economy's structural rate of productivity growth. Heightened uncertainty about the NAIRU in recent years may be closely connected to this productivity acceleration, since such an acceleration may have a temporary disinflationary effect that is reflected in a decline in the "short-run NAIRU" (Meyer, 2000).

rates are set based on policymakers' best estimate of the NAIRU, without regard to the uncertainty surrounding that estimate.[2]

Recently, some authors (Lars Svensson and Michael Woodford, 2000; Swanson, 2000a) have focused attention on the process by which policymakers arrive at the estimate $E_t u^*$. These authors note that, despite the certainty equivalence in (3), in terms of observable variables (u_t, π_t, r_{t-1}, and their lags), optimal policy will in general depend on the level of uncertainty surrounding these indicators. This is because policymakers' problem of estimating u^* is one of signal extraction: in a signal-extraction problem, signal-to-noise ratios of the indicator variables matter.

In this paper, we also focus on policymakers' estimation of u^*. It is standard practice in the literature to assume that policymakers' priors, and all shocks in the model, have a Gaussian distribution. In this special case, policymakers' optimal filtering process for u^* is linear in the observed variables (u_t, π_t, r_{t-1}, and their lags).

However, given the heightened uncertainty facing policymakers in the late 1990's, and the possibility of structural change, it seems natural to think of policymakers as having had beliefs about the NAIRU that, rather than being normally distributed, were instead more diffuse in a region around the mean. In Figure 1, we plot density functions for three distributions that might be used to model policymakers' beliefs about the NAIRU during the early stages of this period. All the distributions have been centered around a mean of 5 for concreteness and comparability. The short-dashed line plots a Gaussian density, which is usually assumed in the literature. By contrast, the solid line plots a uniform density over the interval [4, 6]; it implies a much greater degree of uncertainty about the NAIRU in a region around the mean, although it has the feature that policymakers are absolutely certain the true NAIRU lies neither below 4 nor above 6. The long-dashed line in the figure presents an intermediate case: it has a density that is proportional to $\exp[-0.5(u^* - 5)^4]$. In this paper, we assume that the latter two distributions are more plausible models of policymakers' beliefs and offer a better explanation for policymakers' behavior over this recent historical episode.[3]

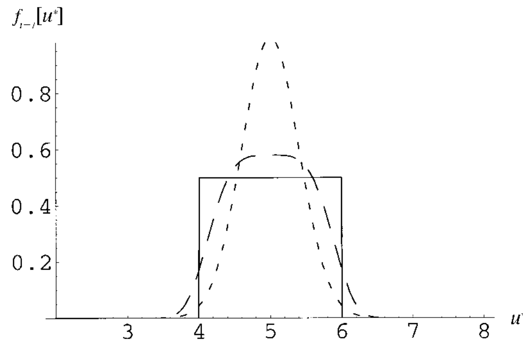

FIGURE 1. SOME POSSIBLE PRIORS ON THE NAIRU

Notes: All the distributions have been centered around a mean of 5. The short-dashed line plots a Gaussian density; the solid line plots a uniform density over the interval [4, 6]; the long-dashed line has a density that is proportional to $\exp[-0.5(u^* - 5)^4]$.

This is not to say, of course, that these distributions will remain fixed forever throughout time. Policymakers will naturally learn about u^* as events unfold, and so their beliefs should be regarded as evolving gradually toward normality, and even narrowing down to a single point, if there are no shocks to u^* and no further structural change.

Swanson (2000b) shows that, given the non-Gaussian prior distributions in Figure 1, optimal updating matches the nonlinear prescriptions put forth by Meyer (1999). This can be seen in Figure 2 (solid lines), in which we present policymakers' optimal updates of u^* in response to realized values of u and π, assuming the uniform distribution from Figure 1 (the results for the long-dashed distribution in Fig. 1 are similar). For comparison, the dashed lines in Figure 2 present the optimal updates for the Gaussian prior from Figure 1. (The dotted lines present our simple nonlinear updating rule, defined below.)

[2] Of course, certainty equivalence fails if multiplicative parameters such as β are included in the estimation (see Wieland, 1998).

[3] One can regard these types of distributions as the result of a structural shift in the economy that is thought to have occurred with some positive probability (Swanson, 2000b).

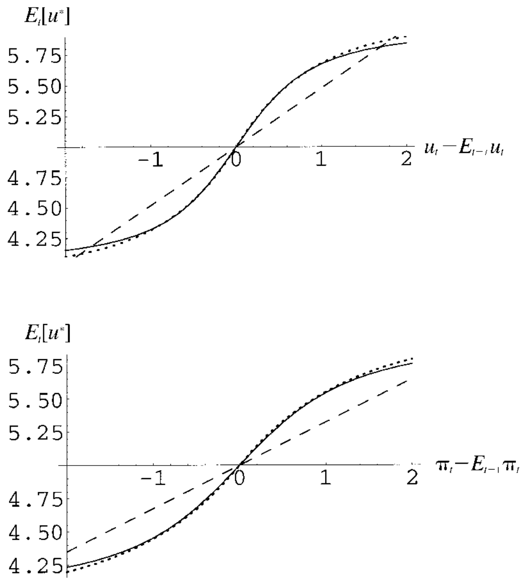

FIGURE 2. OPTIMAL UPDATING, UNIFORM VERSUS GAUSSIAN PRIORS

Notes: The solid lines show policymakers' optimal updates of u^* in response to realized values of u and π, assuming the uniform distribution from Figure 1. The dashed lines present the optimal updates for the Gaussian prior from Figure 1. The dotted lines present the simple nonlinear updating rule.

Note that the optimal updating prescriptions in Figure 2 lead to a nonlinear interest-rate policy in terms of observables (u_t and π_t). This can be seen by plugging the optimal updates from Figure 2 into the estimate for u^* in (3). It follows that policymakers will be very cautious about changing r_t in response to small movements in the unemployment rate (it can also be shown that policymakers will compensate for this by reacting more aggressively to small changes in inflation), but beyond some threshold, they will return to a more active response at the margin.

II. A Simple Nonlinear Updating Rule

The optimal estimate of u^* shown in Figure 2 takes into account realizations of all observable variables in the model. In more complicated and realistic economic models, with a larger number of state variables, indicator variables, and forward-looking expectations, optimal updating with non-Gaussian priors becomes computationally intractable. Moreover, it assumes an unrealistically large amount of knowledge about the structure of the economy on the part of the policymaker. From a practical policy perspective, it is thus more interesting to investigate the performance of a simpler rule that could be studied in more realistic models. Simple rules, such as that in John Taylor (1993), have also been advocated on the grounds that they are robust, that is, perform well across a variety of models (Andrew Levin et al., 1999).

We take as our starting point a Taylor-style rule [equation (3)] and augment it with a simple updating rule that captures the nonlinearities in Figure 2. For comparison, we also consider a linear updating rule, corresponding to the Gaussian prior and dashed lines in Figure 2. Although these simple updating rules will not be optimal in more complicated models, they may have the advantage, like the Taylor rule, of performing well across a variety of models.

We define our linear updating rule as follows (it is optimal for the simple model of Section I, assuming the Gaussian prior of Fig. 1, except that the coefficients have been rounded):

$$(4)\quad \tilde{E}_t u^* = \tilde{E}_{t-1} u^* + 0.5(u_t - \tilde{E}_{t-1} u_t) + 0.35(\pi_t - \tilde{E}_{t-1}\pi_t).$$

The tilde over the E emphasizes the fact that this simple updating rule will not be the true mathematical expectation in more general models.

We build our simple *nonlinear* updating rule ($\hat{E}_t u^*$) as follows. Intuitively, policymakers with the flat-middled prior are very willing to revise their beliefs about u^* near their original point estimate ($E_0 u^*$) but become increasingly more reluctant to do so, at the margin, the more they have revised their estimate of u^* already. We match this qualitative feature by taking a weighted average of two linear rules: near $E_0 u^*$, policymakers use a linear rule that makes them very willing to update estimates of u^* ("open-minded" rule), while further away from $E_0 u^*$, policymakers use a linear rule that makes them much more reluctant to update u^*, at the margin, any further. The weight policymakers place on the reluctant, "stubborn" rule is greater the further is $\hat{E}_{t-1}u^*$ from E_0u^*. Thus,

$$(5)\quad \hat{E}_t u^* = (1 - w_t)\hat{E}_t^{\text{om}} u^* + w_t \hat{E}_t^{\text{s}} u^*$$

where $w_t = \text{Min}\{|\hat{E}_{t-1}u^* - E_0 u^*|, 0.95\}$ and where "om" and "s" refer to the open-minded and stubborn rules, respectively. For the open-minded rule,

$$(6)\quad \hat{E}_t^{\text{om}} u^* = \hat{E}_{t-1}u^* + 2.3[0.5(u_t - \hat{E}_{t-1}u_t) + 0.35(\pi_t - \hat{E}_{t-1}\pi_t)]$$

while for the stubborn rule,

$$(7)\quad \hat{E}_t^{\text{s}} u^* = \hat{E}_{t-1}u^*.$$

The open-minded rule updates 130-percent more aggressively than the linear (Gaussian case) updating rule [equation (4)]. At the other end of the spectrum, the stubborn rule refuses to update any further (this is an extreme case which policymakers never actually reach). The "open-mindedness" and "stubbornness" of these rules, and the weight function, were chosen to match the marginal updating prescriptions from Figure 2. The quality of fit is shown by the dotted line in Figure 2, which graphs our simple nonlinear rule.[4] The approximation is of similarly high quality for combinations of surprises in both unemployment and inflation (not shown).

Finally, before discussing our simulation results, the issue of Kalman filtering, learning, and attenuation in the updating rules should be mentioned. If policymakers' uncertainty about u^* is regenerated every period (e.g., by shocks to u^*), then the coefficients in these updating rules would be nearly optimal for all future periods, and we could regard these updating rules as constant over time. However, we find it more realistic and interesting to assume that policymakers gradually learn about the true value of u^* over time. Thus, policymakers will update by less and less each period as the cumulative number of observations increases. We calculate what this rate of attenuation is for optimal (Kalman) filtering for the Gaussian prior, approximate it with a geometric decay, and apply this rate of attenuation to both our simple linear and nonlinear updating rule coefficients over time.

III. Simulations and Results

Having defined our simple linear and nonlinear updating rules, we now test their performance in model simulations. We consider two models: the simple backward-looking model of Section I, and the forward-looking model of Glenn Rudebusch (2000).[5] For each simulation, we fix a true value for u^* and calculate the losses that result from following the simple linear and nonlinear policies of Section II (this requires a nonlinear rational-expectations equation-solver for the forward-looking model). Losses in each period are the equal-weighted sum of squared inflation deviations from the target and squared unemployment deviations from u^*. For each true value of u^*, we run 10,000 simulations and calculate the average loss at each point in time.

Figure 3 presents the results, at a horizon of four quarters, for the simple model of Section I (similar results for Rudebusch's model are reported in Meyer et al. [2001]). The most important feature of Figure 3 is the intersection of the loss schedules for the linear and nonlinear policies. Neither rule completely dominates the other. For large deviations of the true NAIRU from policymakers' prior mean of 5 (the left and

[4] The simple rule is graphed as the sum of small marginal updates, using equation (5) at each step. The optimal rule, a function of the one-period surprise $u_t - E_{t-1}u_t$ (and/or $\pi_t - E_{t-1}\pi_t$), is most naturally thought of at a lower frequency, in which the policymaker is looking back over a longer period of time and updating by a greater amount (Swanson, 2000b). In our higher-frequency (quarterly) simulations, surprises in unemployment and inflation are small, so the sum of small marginal updates captures very well policymakers' revision process.

[5] This model is essentially a weighted average of the model of Rudebusch and Svensson (1999) and the forward-looking "new Keynesian consensus" model favored by many recent authors (e.g., Richard Clarida et al., 1999). This feature allows us to assess the rules' performance for different degrees of forward-lookingness in the economy. We use an Okun's Law of 2:1 to map output gaps to unemployment gaps in the model.

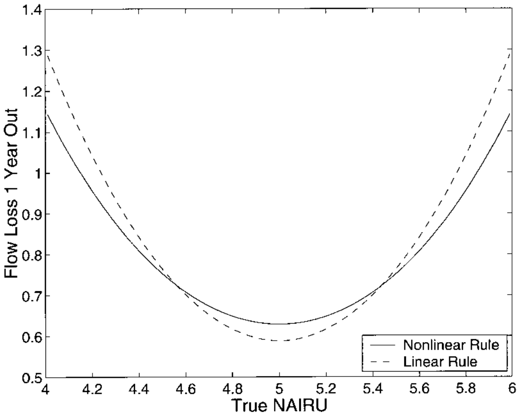

FIGURE 3. LOSSES FROM NONLINEAR AND LINEAR POLICY RULES

right extremes of Fig. 3), the simple nonlinear rule updates very aggressively and performs substantially better than the simple linear rule, because policymakers tend to arrive more quickly at a correct estimate for u^*. For small deviations of the true NAIRU from policymakers' prior mean (the middle region of Fig. 3), the simple nonlinear rule updates aggressively, and incorrectly, in response to transitory shocks to u and π and thus performs more poorly than the simple linear rule.

Which rule should be followed thus depends on policymakers' uncertainty about u^*. If policymakers are not particularly uncertain about u^*, then the simple nonlinear rule seems to offer few benefits and, in fact, even offers worse performance than the linear rule. However, if policymakers are very uncertain about the true value of u^* and consider a range of values fairly likely, the simple nonlinear rule offers greater robustness with respect to this uncertainty.

IV. Conclusions

The Taylor rule was initially offered as a normative prescription. That is, it was designed to have excellent stabilization properties that were robust across different models and across a range of shocks to the economy. Subsequently, Taylor (1993) concluded that the rule also had descriptive or predictive value; that is, it has described the way monetary policy has been conducted, at least over the previous decade.

The nonlinear policy rule developed in this paper may have the same two properties. First, we have shown that a simple nonlinear rule might outperform a linear rule during a period of heightened uncertainty about the NAIRU; this is its normative value. This result must nevertheless be viewed as only suggestive and preliminary, as it is based on only two very simple macroeconometric models. Further work will be necessary to determine how robust the results are across different models, as well as in larger, more fully specified models.

Second, the nonlinear rule may describe the policies followed during this period better than the linear rule. There are several features of recent policy that seem to conform to the spirit of the nonlinear rule. First, the Federal Reserve Board staff has continuously updated its estimate of the NAIRU (specifically the short-run NAIRU) during this period, and the updated estimate of the NAIRU likely influenced to some degree the decisions of some members of the Federal Open Market Committee (FOMC). Second, it appears that policymakers, for a while, were relatively tolerant of declines in the unemployment rate in a region around their initial prior—in effect, attenuating their response to changes in the unemployment rate. Third, when the unemployment rate fell to the lower end of a plausible range for the NAIRU, monetary policy moved preemptively and aggressively to slow growth and thereby reduce the risk of higher inflation.

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