The Bond Premium in a DSGE Model with Long-Run Real and Nominal Risks

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Outline

1. Motivation and Background
2. Epstein-Zin Preferences in a Standard NK Model
3. Long-Run Risks
4. Model Implications
5. Conclusions

The Bond Premium Puzzle

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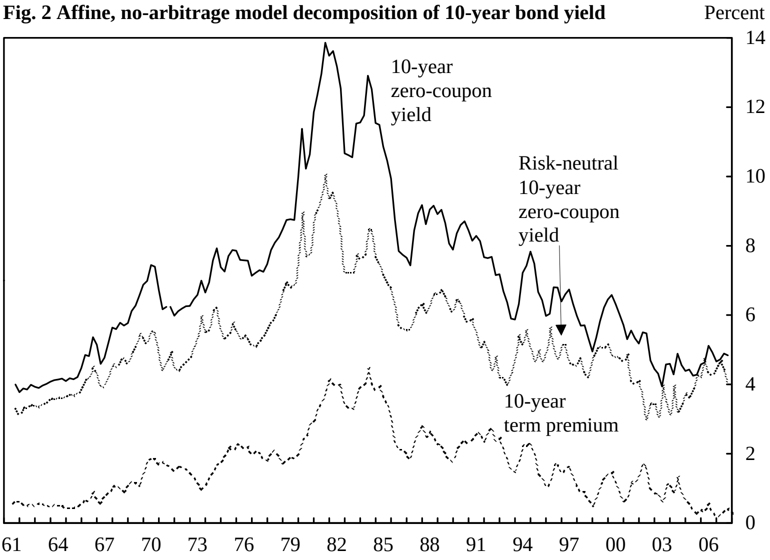
The UIP premium puzzle: excess returns on high-interest-rate foreign currencies are much larger (and more variable) than can be explained by standard preferences in a DSGE model.

Kim-Wright Term Premium



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Relative to equity premium, the term premium:

- applies to a larger volume of securities
- is used by central banks to measure expectations of monetary policy, inflation
- only requires modeling short-term interest rate, not dividends or leverage
- provides an additional perspective on the model
- tests nominal rigidities

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More generally:

- many empirical questions about risk premia require a structural DSGE model to provide reliable answers
- DSGE models widely used in macroeconomics; total failure to explain risk premia may signal flaws in the model

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We examine to what extent the Piazzesi-Schneider results generalize to the DSGE case

Related Strands of the Literature

The Bond Premium in a DSGE Model:

- Backus-Gregory-Zin (1989), Donaldson-Johnson-Mehra (1990), Den Haan (1995), Doh (2006), Rudebusch-Swanson (2008)

Epstein-Zin Preferences and the Bond Premium in an Endowment Economy:

- Piazzesi-Schneider (2006), Colacito-Croce (2007), Backus-Routledge-Zin (2007), Gallmeyer-Hollifield-Palomino-Zin (2007), Bansal-Shaliastovich (2008), Doh (2008)

Epstein-Zin Preferences in a DSGE Model:

- Tallarini (2000), Croce (2007), Levin-Lopez-Salido-Nelson-Yun (2008)

Epstein-Zin Preferences and the Bond Premium in a DSGE Model:

- van Binsbergen-Fernandez-Villaverde-Koijen-Rubio-Ramirez (2008)

Epstein-Zin Preferences in a Standard DSGE Model

2. Epstein-Zin Preferences in a Standard NK Model
   - Epstein-Zin Preferences
   - Standard New Keynesian Model
   - Price Assets in the Model
   - Solve the Model
   - Results

Epstein-Zin Preferences

Standard preferences:

$$V_t \equiv u(c_t, l_t) + \beta E_t V_{t+1}$$

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- or $u \leq 0$ and $V_t \equiv u(c_t, l_t) - \beta \left(E_t (-V_{t+1})^{1-\alpha} \right)^{1/(1-\alpha)}$

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We'll use standard NK utility kernel:

$$u(c_t, l_t) \equiv \frac{c_t^{1-\gamma}}{1-\gamma} - \chi_0 \frac{l_t^{1+\chi}}{1+\chi},$$

Epstein-Zin Preferences

Household optimality conditions with EZ preferences:

$$\mu_t \, u_1\big|_{(c_t, l_t)} = P_t \lambda_t$$

$$-\mu_t \, u_2\big|_{(c_t, l_t)} = w_t \lambda_t$$

$$\lambda_t = \beta E_t \lambda_{t+1}(1 + r_{t+1})$$

$$\mu_t = \mu_{t-1} \left(E_{t-1} V_t^{1-\alpha} \right)^{\alpha/(1-\alpha)} V_t^{-\alpha}, \quad \mu_0 = 1$$

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Recall: $V_t = u(c_t, l_t) + \beta\big(E_t V_{t+1}^{1-\alpha}\big)^{1/(1-\alpha)}$

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Recall: $V_t = u(c_t, l_t) + \beta \left(E_t V_{t+1}^{1-\alpha} \right)^{1/(1-\alpha)}$

Stochastic discount factor:

$$m_{t,t+1} \equiv \frac{\beta u_1\big|_{(c_{t+1},l_{t+1})}}{u_1\big|_{(c_t,l_t)}} \left(\frac{V_{t+1}}{\left(E_t V_{t+1}^{1-\alpha} \right)^{1/(1-\alpha)}} \right)^{-\alpha} \frac{P_t}{P_{t+1}}$$

New Keynesian Model (Very Standard)

Continuum of differentiated firms:

- face Dixit-Stiglitz demand with elasticity $\frac{1+\theta}{\theta}$, markup θ
- set prices in Calvo contracts with avg. duration 4 quarters
- identical production functions $y_t = A_t \bar{k}^{1-\eta} l_t^{\eta}$
- have firm-specific capital stocks
- face aggregate technology $\log A_t = \rho_A \log A_{t-1} + \varepsilon_t^A$

Parameters $\theta = .2$, $\rho_A = .9$, $\sigma_A^2 = .01^2$

Perfectly competitive goods aggregation sector

New Keynesian Model (Very Standard)

Government:

- imposes lump-sum taxes G_t on households
- destroys the resources it collects
- $\log G_t = \rho_G \log G_{t-1} + (1 - \rho_g) \log \bar{G} + \varepsilon_t^G$

Parameters $\bar{G} = .17\bar{Y}$, $\rho_G = .9$, $\sigma_G^2 = .004^2$

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Monetary Authority:

$$i_t = \rho_i i_{t-1} + (1 - \rho_i) \left[1/\beta + \pi_t + g_y(y_t - \bar{y}) + g_\pi(\bar{\pi}_t - \pi^*) \right] + \varepsilon_t^i$$

Parameters $\rho_i = .73$, $g_y = .53$, $g_\pi = .93$, $\pi^* = 0$, $\sigma_i^2 = .004^2$

Asset Pricing

Asset pricing:

$$p_t = d_t + E_t[m_{t+1} p_{t+1}]$$

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Zero-coupon bond pricing:

$$p_t^{(n)} = E_t[m_{t+1} p_{t+1}^{(n-1)}]$$

$$i_t^{(n)} = -\frac{1}{n} \log p_t^{(n)}$$

Notation: let $i_t \equiv i_t^{(1)}$

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$$\widetilde{p}_t^{(n)} = 1 + \delta_c \, E_t m_{t+1} \widetilde{p}_{t+1}^{(n)}$$

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Term premium:

$$\psi_t^{(n)} \equiv \log\left(\frac{\delta_c \widetilde{p}_t^{(n)}}{\widetilde{p}_t^{(n)} - 1}\right) - \log\left(\frac{\delta_c \widehat{p}_t^{(n)}}{\widehat{p}_t^{(n)} - 1}\right)$$

Solving the Model

The standard NK model above has a relatively large numer of state variables: A_{t-1}, G_{t-1}, i_{t-1}, Δ_{t-1}, $\bar{\pi}_{t-1}$, ε_t^A, ε_t^G, ε_t^i

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We solve the model by approximation around the nonstochastic steady state (perturbation methods)

- In a first-order approximation, term premium is zero
- In a second-order approximation, term premium is a constant (sum of variances)
- So we compute a *third*-order approximation of the solution around nonstochastic steady state
- Perturbation AIM algorithm in Swanson, Anderson, Levin (2006) quickly computes *n*th order approximations

Empirical and Model-Based Unconditional Moments

Variable	U.S. Data	EU Preferences	EZ Preferences	"best fit" EZ Preferences
sd[C]	1.19	1.40	1.46	2.12
sd[L]	1.71	2.48	2.50	1.89
sd[w^r]	0.82	2.02	2.02	2.02
sd[π]	2.52	2.22	2.30	2.96
sd[i]	2.71	1.86	1.93	2.65
sd[$i^{(40)}$]	2.41	0.52	0.57	1.17
mean[$\psi^{(40)}$]	1.06	.010	.438	1.06
sd[$\psi^{(40)}$]	0.54	.000	.053	.162
mean[$i^{(40)} - i$]	1.43	$-$.038	.390	0.95
sd[$i^{(40)} - i$]	1.33	1.41	1.43	1.59
mean[$x^{(40)}$]	1.76	.010	.431	1.04
sd[$x^{(40)}$]	23.43	6.52	6.87	10.77
memo: IES		.5	.5	.5
quasi-CRRA		2	75	90

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$$\text{CRRA} \quad = \quad \frac{-W\,V''(W)}{V'(W)} + \alpha\,\frac{W\,V'(W)}{V(W)}$$

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$$\begin{aligned} \text{CRRA} &= \frac{-W\,V''(W)}{V'(W)} + \alpha\,\frac{W\,V'(W)}{V(W)} \\ &= \frac{-u_{11} + \lambda u_{12}}{u_1}\frac{c}{1+w\lambda} + \alpha\frac{c\,u_1}{u} \end{aligned}$$

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see “Risk Aversion, the Labor Margin, and Asset Pricing in a DSGE Model”

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- Barillas-Hansen-Sargent (2008):

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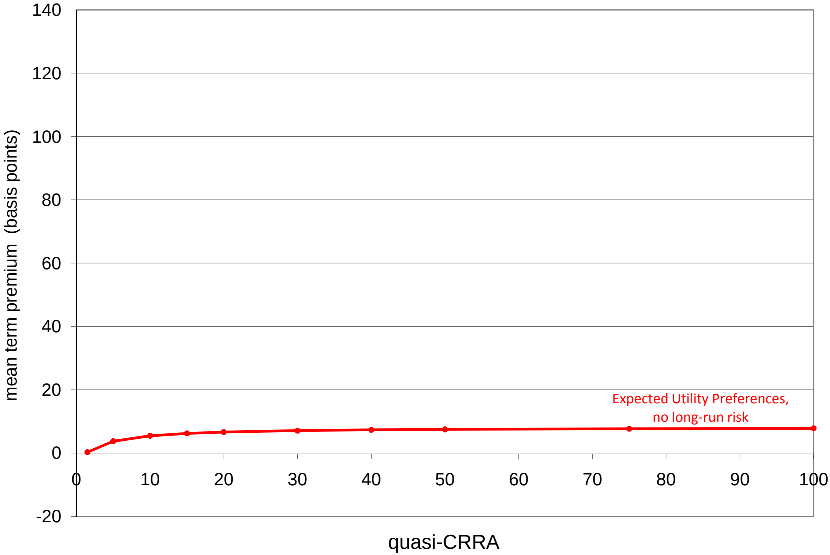
- Guvenen (2006), Moskowitz-Vissing-Jorgensen (2009): heterogeneous agents

Risk Aversion and the Term Premium



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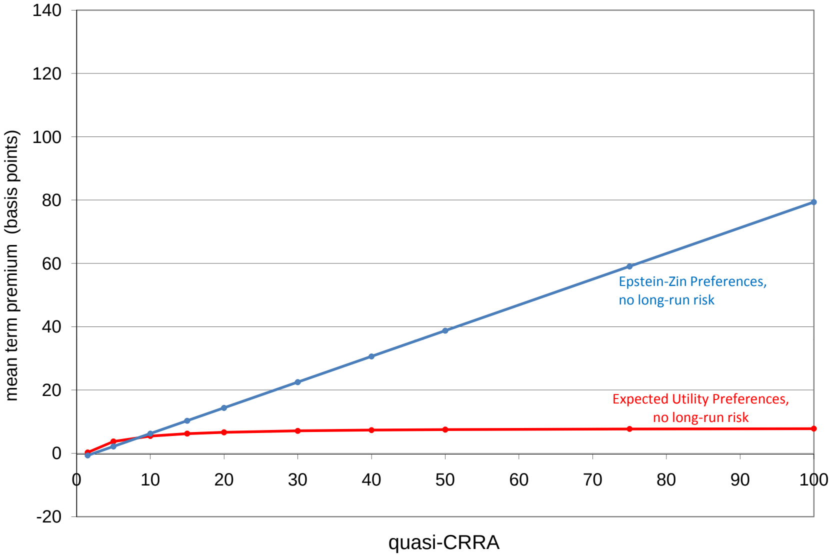

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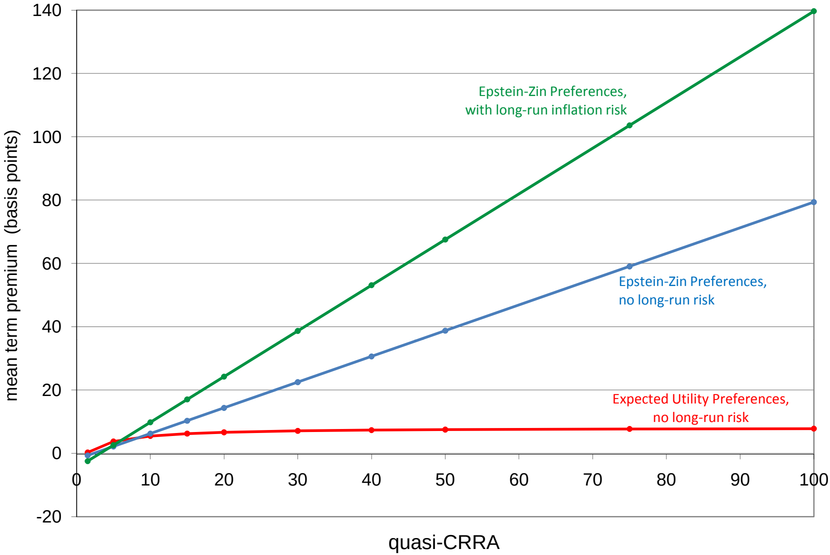

Long-Run Risks

3 Long-Run Risks
- Long-Run Inflation Risk
- Long-Run Real Risk

Long-Run Inflation Risk

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Long-run inflation risk makes long-term bonds more risky:

- same idea as Bansal-Yaron (2004), but with nominal risk rather than real risk
- long-term inflation expectations more observable than long-term consumption growth

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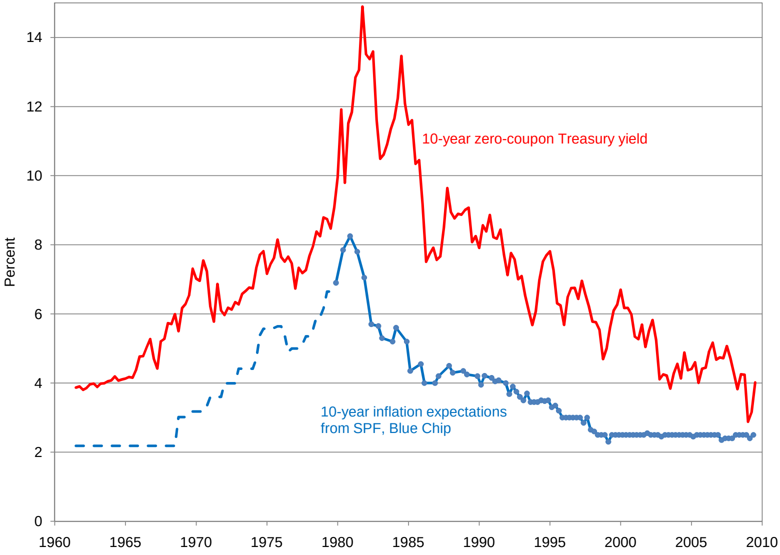

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- long-term inflation expectations more observable than long-term consumption growth
- other evidence (Kozicki-Tinsley, 2003, Gürkaynak, Sack, Swanson, 2005) that long-term inflation expectations in the U.S. vary

Long-Run Inflation Risk

Suppose:

$$\pi_t^* = \rho_\pi^* \pi_{t-1}^* + \varepsilon_t^{\pi^*}$$

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Then:

- inflation is volatile, but not risky
- in fact, long-term bonds act like insurance:
  when $\pi^* \uparrow$, then $C \uparrow$ and $p^{(40)} \downarrow$
- result: term premium is *negative*

Long-Run Inflation Risk

Consider instead:

$$\pi_t^* = \rho_\pi^* \pi_{t-1}^* + (1 - \rho_\pi^*)\theta_{\pi^*}(\overline{\pi}_t - \pi_t^*) + \varepsilon_t^{\pi^*}$$

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Consider instead:

$$\pi_t^* = \rho_\pi^* \pi_{t-1}^* + (1 - \rho_\pi^*)\theta_{\pi^*}(\overline{\pi}_t - \pi_t^*) + \varepsilon_t^{\pi^*}$$

- θ_{π^*} describes pass-through from current π to long-term π^*
- Gürkaynak, Sack, and Swanson (2005) found evidence for $\theta_{\pi^*} > 0$ in U.S. bond response to macro data releases
- makes long-term bonds act less like insurance:
  when technology/supply shock, then $\pi \uparrow$, $C \downarrow$, and $p^{(40)} \downarrow$
  supply shocks become very costly
- The term premium is *positive*, closely associated with θ_{π^*}

Model-Based Moments with Long-Run Inflation Risk

Variable	U.S. Data	EU Preferences & LR π^* Risk	EZ Preferences & LR π^* Risk
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sd[$x^{(40)}$]	23.43	13.07	11.64
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quasi-CRRA		2	90

Long-Run Productivity Risk

Following Bansal and Yaron (2004), introduce long-run real risk to make the economy more risky:

Assume productivity follows:

$$\log A_t = \log A_t^* + \varepsilon_t^A$$

$$\log A_t^* = \rho_{A^*} \log A_{t-1}^* + \varepsilon_t^{A^*}$$

- makes the economy much riskier to agents
- increases volatility of stochastic discount factor

Model-Based Moments w/Long-Run Productivity Risk

Variable	U.S. Data	EZ Preferences & LR π^* Risk	EZ Preferences & LR A^* risk
sd[C]	1.19	2.01	2.37
sd[L]	1.71	1.37	2.13
sd[w^r]	0.82	1.52	1.81
sd[π]	2.52	3.25	2.95
sd[i]	2.71	2.94	2.86
sd[$i^{(40)}$]	2.41	1.89	1.66
mean[$\psi^{(40)}$]	1.06	1.05	0.98
sd[$\psi^{(40)}$]	0.54	0.51	0.28
mean[$i^{(40)} - i$]	1.43	0.96	0.89
sd[$i^{(40)} - i$]	1.33	1.10	1.36
mean[$x^{(40)}$]	1.76	1.04	0.96
sd[$x^{(40)}$]	23.43	11.64	12.20
memo: IES		1.1	.5
quasi-CRRA		90	90

Model Implications

4 Model Implications

- Nominal Yield Curve is Upward-Sloping
- Term Premium is Countercylical
- Model Is Nonhomothetic, Heteroskedastic

Nominal Yield Curve is Upward-Sloping

Backus-Gregory-Zin (1989), Den Haan (1995)

- if interest rates are low in recessions
- then bond prices rise in recessions
- $\Longrightarrow$ the term premium should be negative
- the yield curve slopes downward

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- technology shocks imply that inflation is high in recessions
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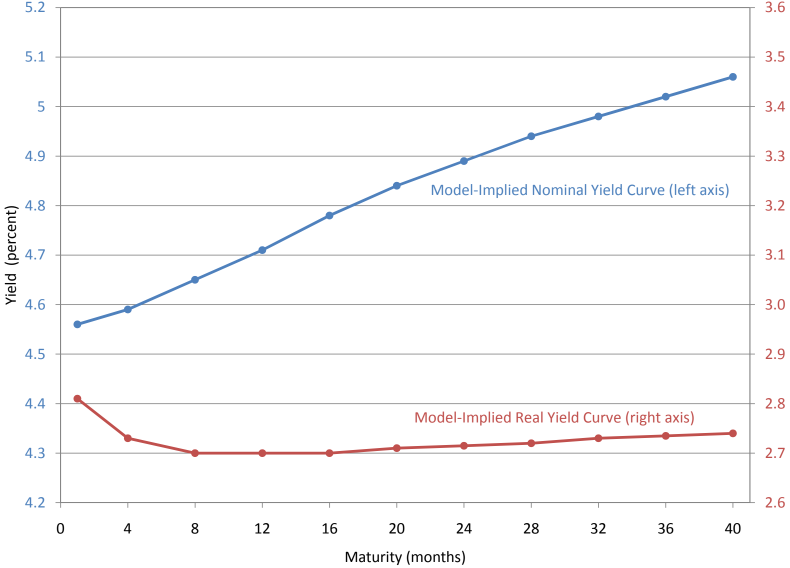
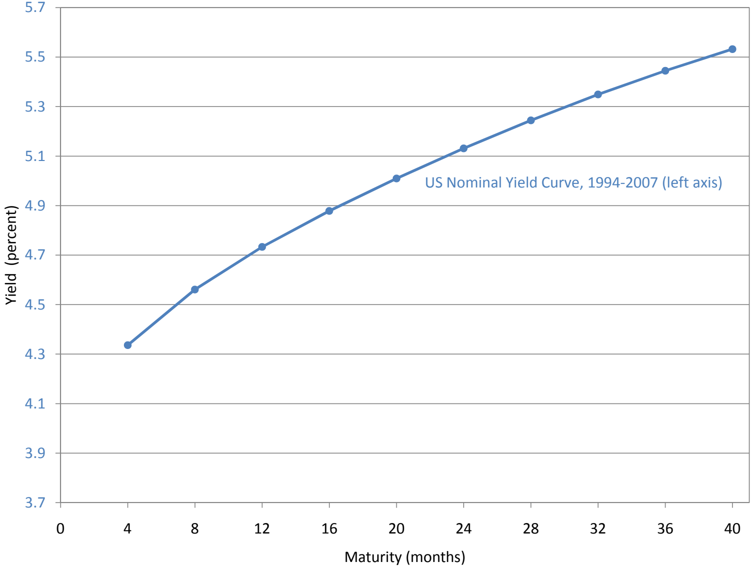
Note: Backus et. al intuition still applies to real yield curve

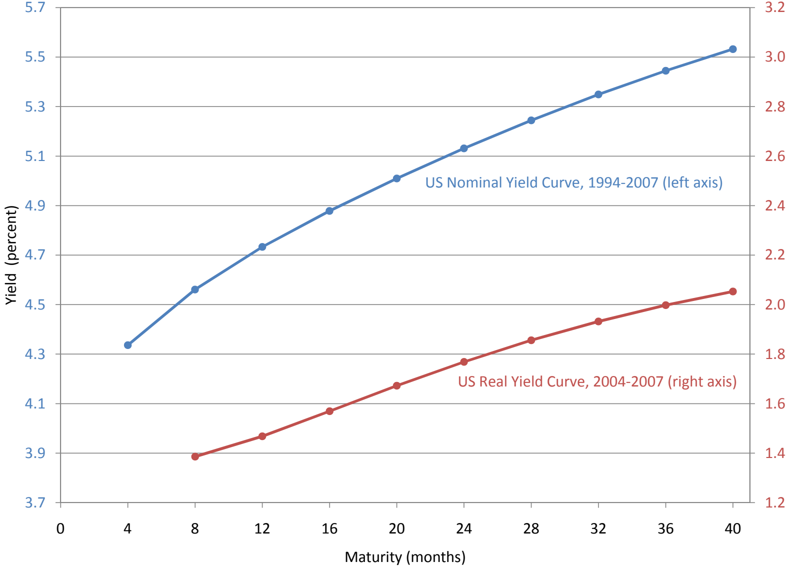
Nominal Yield Curve is Upward-Sloping



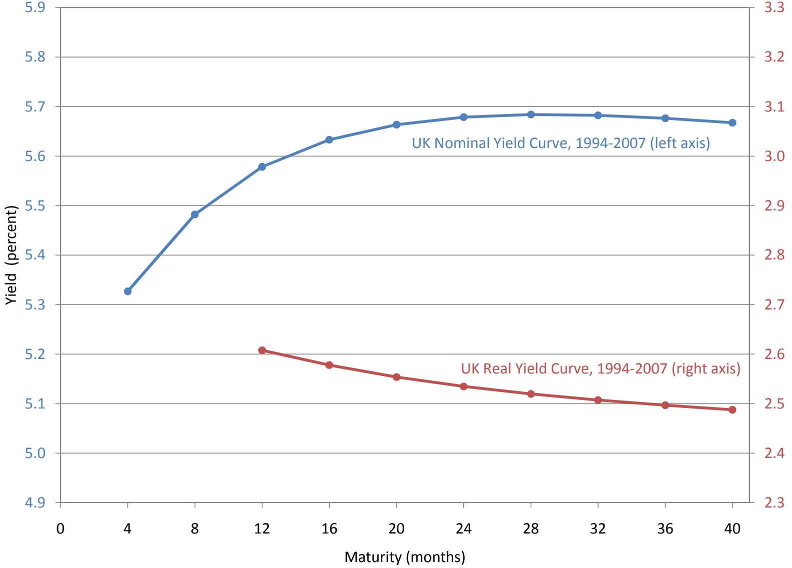
US Yield Curve, 1994–2007



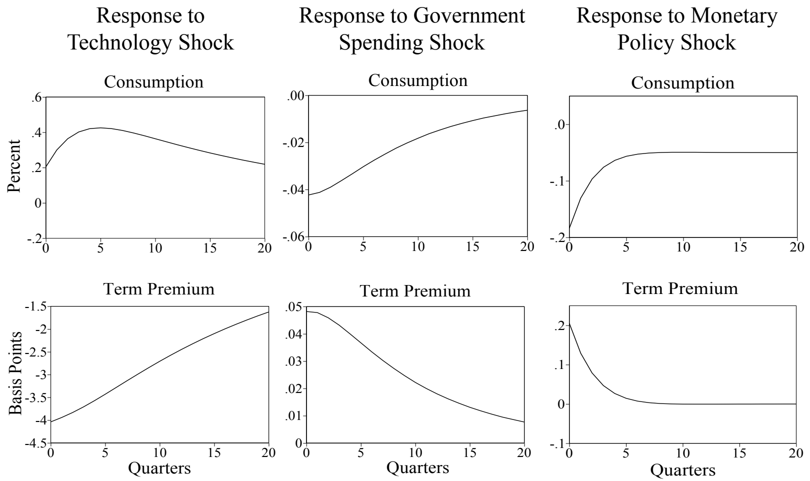
US Yield Curve, 1994–2007



UK Yield Curve, 1994–2007



Model Term Premium is Countercylical



Model Is Nonhomothetic, Heteroskedastic

$$p_t^{(2)} - \hat{p}_t^{(2)} = E_t m_{t+1} p_{t+1}^{(1)} - E_t m_{t+1} E_t p_{t+1}^{(1)} = Cov_t(m_{t+1}, p_{t+1}^{(1)})$$

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Second-order solution:

$$\begin{aligned} x_t &= \mu_x + \sum \alpha_x dx_{t-1} + \sum \alpha_\varepsilon \varepsilon_t \\ &\quad + \sum \alpha_{xx} dx_{t-1} dx_{t-1} + \sum \alpha_{x\epsilon} dx_{t-1} \varepsilon_t + \sum \alpha_{\varepsilon\varepsilon} \varepsilon_t \varepsilon_t + \ldots \end{aligned}$$

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Model	term premium mean (bp)	term premium std dev (bp)
baseline model	86.5	11.0

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baseline model	86.5	11.0
log-linear log-normal	86.5	0.0

Conclusions

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3. Epstein-Zin preferences can solve bond premium puzzle in endowment economy, are much more promising in NK DSGE framework:
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4. Long-run risks reduce the required quasi-CRRA, increase volatility of risk premia, help fit financial moments