Risk Aversion and the Labor Margin in Dynamic Equilibrium Models

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Coefficient of Relative Risk Aversion

Suppose a household has preferences:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t),$$

$$u(c_t, l_t) = \frac{c_t^{1-\gamma}}{1-\gamma} - \eta\, l_t$$

What is the household's coefficient of relative risk aversion?

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What is the household's coefficient of relative risk aversion?

Answer: 0

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What is the household's coefficient of relative risk aversion?

Answer: $\dfrac{1}{\frac{1}{\gamma} + \frac{1}{\chi}}$

Outline of Presentation

- Define risk aversion rigorously in dynamic equilibrium models
- Derive closed-form expressions
- Show the labor margin can have big effects on risk aversion
- Compute numerical solutions far away from steady state
- Relate risk aversion to asset pricing, stochastic discount factor

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See the paper for:

- Epstein-Zin preferences
- internal, external habits

A Household

Household preferences:

$$E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} u(c_\tau, l_\tau),$$

Flow budget constraint:

$$a_{\tau+1} = (1 + r_\tau) a_\tau + w_\tau l_\tau + d_\tau - c_\tau,$$

No-Ponzi condition:

$$\lim_{T\to\infty} \prod_{\tau=t}^{T} (1 + r_{\tau+1})^{-1} a_{T+1} \geq 0,$$

$\{w_\tau, r_\tau, d_\tau\}$ are exogenous processes, governed by θ_τ

The Value Function

State variables of the household's problem are $(a_t; \theta_t)$.

Let:

$$c_t^* \equiv c^*(a_t; \theta_t),$$

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Value function, Bellman equation:

$$V(a_t; \theta_t) = u(c_t^*, l_t^*) + \beta E_t V(a_{t+1}^*; \theta_{t+1}),$$

where:

$$a_{t+1}^* \equiv (1 + r_t)a_t + w_t l_t^* + d_t - c_t^*.$$

Technical Conditions

Assumption 1. *The function $u(c_t, l_t)$ is increasing in its first argument, decreasing in its second, twice-differentiable, and strictly concave.*

Assumption 2. *The value function $V : X \to \mathbb{R}$ for the household's optimization problem exists and satisfies the Bellman equation*

$$V(a_t; \theta_t) = \max_{(c_t, l_t) \in \Gamma(a_t; \theta_t)} u(c_t, l_t) + \beta E_t V(a_{t+1}; \theta_{t+1}).$$

Assumption 3. *For any $(a_t; \theta_t) \in X$, the household's optimal choice (c_t^*, l_t^*) lies in the interior of $\Gamma(a_t; \theta_t)$.*

Assumption 4. *The value function $V(\cdot; \cdot)$ is twice-differentiable in its first argument. (It then follows that c^*, l^* are differentiable.)*

Assumptions about the Economic Environment

Assumption 5. *The household is* atomistic*.*

Assumption 6. *The household is* representative*.*

Assumption 7. *The model has a* nonstochastic steady state*,* $x_t = x_{t+k}$ *for* $k = 1, 2, \ldots,$ *and* $x \in \{c, l, a, w, r, d, \theta\}$*.*

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Assumption 7′. *The model has a* balanced growth path *that can be renormalized to a nonstochastic steady state after a suitable change of variables.*

Arrow-Pratt in a Static One-Good Model (Review)

Compare:

$$E\,u(c + \sigma\varepsilon) \quad \text{vs.} \quad u(c - \mu)$$

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$$u(c-\mu) \;\approx\; u(c) - \mu u'(c),$$

$$E\,u(c+\sigma\varepsilon) \;\approx\; u(c) + u'(c)\sigma E[\varepsilon] + \frac{1}{2}u''(c)\sigma^2 E[\varepsilon^2],$$

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Coefficient of absolute risk aversion is defined to be:

$$\lim_{\sigma\to 0} 2\mu(\sigma)/\sigma^2 \;=\; \frac{-u''(c)}{u'(c)}\,.$$

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Note $(*)$ is equivalent to gamble over income:

$$a_{t+1} = (1 + r_t)a_t + w_t l_t + (d_t + \sigma\varepsilon_{t+1}) - c_t,$$

or asset returns:

$$a_{t+1} = (1 + r_t + \sigma\tilde{\varepsilon}_{t+1})a_t + w_t l_t + d_t - c_t.$$

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Welfare loss from μ:

$$V_1(a_t; \theta_t) \frac{\mu}{(1 + r_t)}$$

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Loss from σ:

$$\beta E_t V_{11}(a^*_{t+1}; \theta_{t+1})\frac{\sigma^2}{2}.$$

Coefficient of Absolute Risk Aversion

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Proposition 1. *The household's coefficient of absolute risk aversion at* $(a_t; \theta_t)$ *satisfies*

$$R^a(a_t; \theta_t) \;=\; \frac{-\,E_t V_{11}(a^*_{t+1}; \theta_{t+1})}{E_t V_1(a^*_{t+1}; \theta_{t+1})}\,.$$

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Evaluated at the nonstochastic steady state, this simplifies to:

$$R^a(a; \theta) \;=\; \frac{-V_{11}(a; \theta)}{V_1(a; \theta)} \,.$$

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Solve for V_1 and V_{11}

Benveniste-Scheinkman:

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Differentiate $(*)$ to get:

$$V_{11}(a_t;\theta_t) \;=\; (1+r_t)\left[u_{11}(c_t^*, l_t^*)\,\frac{\partial c_t^*}{\partial a_t} \;+\; u_{12}(c_t^*, l_t^*)\,\frac{\partial l_t^*}{\partial a_t}\right].$$

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Household Euler equation:

$$u_1(c_t^*, l_t^*) \;=\; \beta E_t (1 + r_{t+1})\, u_1(c_{t+1}^*, l_{t+1}^*),$$

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Differentiate, substitute out for $\partial l_t^* / \partial a_t$, and use BC, TVC to get:

$$\frac{\partial c_t^*}{\partial a_t} = \frac{r}{1 + w\lambda} \, .$$

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Proposition 2. *The household's coefficient of absolute risk aversion in Proposition 1, evaluated at steady state, satisfies:*

$$R^a(a;\theta) \;=\; \frac{-V_{11}(a;\theta)}{V_1(a;\theta)} \;=\; \frac{-u_{11} + \lambda u_{12}}{u_1}\,\frac{r}{1+w\lambda}\,.$$

Coefficient of Absolute Risk Aversion

Corollary 3.

$$R^a(a;\theta) = \frac{-u_{11} + \lambda u_{12}}{u_1} \frac{r}{1 + w\lambda} \leq \frac{-u_{11}}{u_1} r \, .$$

If $r < 1$, then $R^a(a;\theta)$ is also less than $-u_{11}/u_1$.

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$$u(c_t, l_t) = c_t^{\theta}(\bar{l} - l_t)^{1-\theta}.$$

Relative Risk Aversion

Consider Arrow-Pratt gamble of general size A_t:

$$a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t + A_t \sigma \varepsilon_{t+1},$$

vs.

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Risk aversion coefficient for this gamble:

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A natural benchmark for A_t is household wealth at time t.

Household Wealth

In DSGE framework, household wealth has more than one component:

- financial assets a_t
- present value of labor income, $w_t l_t$
- present value of net transfers, d_t
- present value of leisure, $w_t(\bar{l} - l_t)$?

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Leisure, in particular, can be hard to define, e.g.,

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Different definitions of household wealth lead to different definitions of relative risk aversion.

Two Coefficients of Relative Risk Aversion

Definition 2. *The coefficient of relative risk aversion, $R^r(a_t;\theta_t)$, is given by $(*)$, with $A_t \equiv (1+r_t)^{-1} E_t \sum_{\tau=t}^{\infty} m_{t,\tau}\left(c_\tau^* + w_\tau(\bar{l} - l_\tau^*)\right)$.*

In steady state:

$$R^r(a;\theta) = \frac{-A\,V_{11}(a;\theta)}{V_1(a;\theta)} = \frac{-u_{11} + \lambda u_{12}}{u_1}\,\frac{c + w(\bar{l} - l)}{1 + w\lambda}.$$

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Definition 3. *The consumption-only coefficient of relative risk aversion, $R^c(a_t;\theta_t)$, is given by $(*)$, with $\tilde{A}_t \equiv (1+r_t)^{-1} E_t \sum_{\tau=t}^{\infty} m_{t,\tau} c^*_\tau$.*

In steady state:

$$R^c(a;\theta) = \frac{-\tilde{A}\, V_{11}(a;\theta)}{V_1(a;\theta)} = \frac{-u_{11} + \lambda u_{12}}{u_1} \frac{c}{1 + w\lambda}.$$

Risk Aversion and the IES

Corollary 5.

i) $R^c(a;\theta)$ *and the intertemporal elasticity of substitution are reciprocal if and only if* $\lambda = 0$*;*

ii) $R^r(a;\theta)$ *and the intertemporal elasticity of substitution are reciprocal if and only if* $\lambda = (\bar{l} - l)/c$*.*

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i) $R^c(a;\theta)$ and the intertemporal elasticity of substitution are reciprocal if and only if $\lambda = 0$;

ii) $R^r(a;\theta)$ and the intertemporal elasticity of substitution are reciprocal if and only if $\lambda = (\bar{l} - l)/c$.

Proof:

$$IES = \frac{-u_1}{u_{11} - \lambda u_{12}} \frac{1}{c}$$

$$R^c(a;\theta) = \frac{-u_{11} + \lambda u_{12}}{u_1} \frac{c}{1 + w\lambda}$$

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Example 1

Period utility function: $$u(c_t, l_t) = \frac{\left(c_t^{\chi}(1 - l_t)^{1-\chi}\right)^{1-\gamma}}{1 - \gamma}$$

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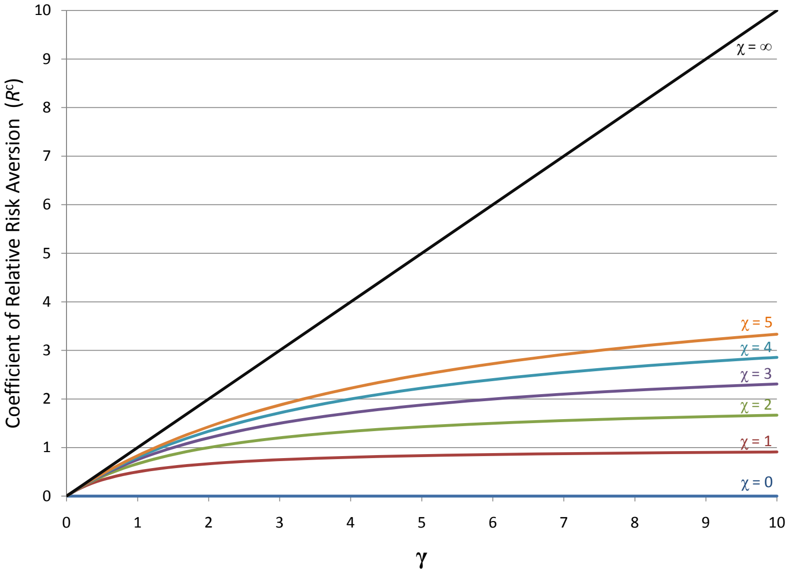
$R^c(a;\theta) = \frac{\gamma}{1 + \frac{\gamma}{\chi}\frac{wl}{c}} \approx \frac{1}{\frac{1}{\gamma} + \frac{1}{\chi}}$, using $c = wl + ra + d \approx wl$.

Example 2


Coefficient of Relative Risk Aversion (R^c)
γ
χ = ∞
χ = 5
χ = 4
χ = 3
χ = 2
χ = 1
χ = 0
0
1
2
3
4
5
6
7
8
9
10

Risk Aversion Away from the Steady State

$$u(c_t, l_t) = \frac{c_t^{1-\gamma}}{1-\gamma} - \eta \frac{l_t^{1+\chi}}{1+\chi}, \qquad \gamma = 2,\ \chi = 1.5$$

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Standard RBC model:

$$Y_t = A_t K_t^{1-\alpha} L_t^{\alpha}$$

$$K_{t+1} = (1-\delta) K_t + Y_t - C_t$$

$$C_t^{-\gamma} = \beta E_t (1+r_{t+1}) C_{t+1}^{-\gamma}$$

$$\eta L_t^{\chi} / C_t^{-\gamma} = w_t$$

$$r_t = (1-\alpha) Y_t / K_t - \delta$$

$$w_t = \alpha Y_t / L_t$$

$$\log A_t = \rho \log A_{t-1} + \varepsilon_t$$

Risk Aversion Away from the Steady State

Auxiliary equations:

$$R^a(a_t;\theta_t) = \frac{-E_t V_{11}(a^*_{t+1};\theta_{t+1})}{E_t V_1(a^*_{t+1};\theta_{t+1})}$$

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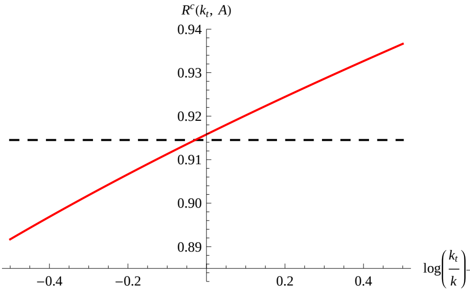
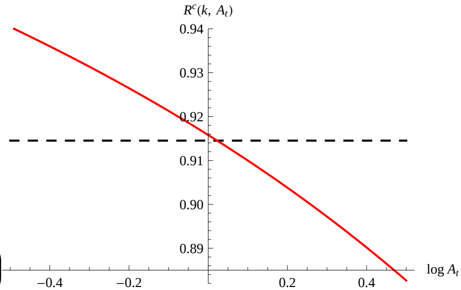
Risk Aversion Away from the Steady State

Numerical solution to the model:





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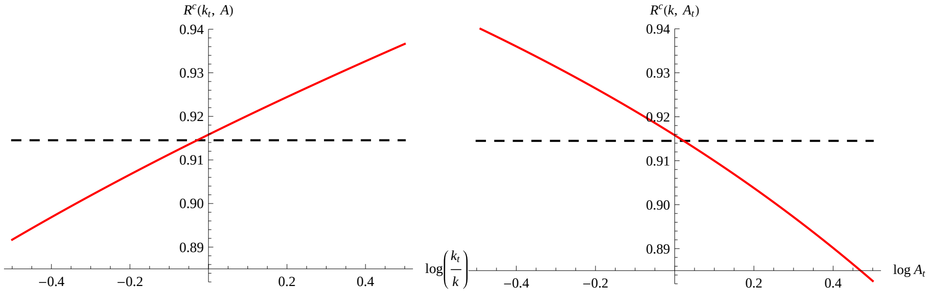
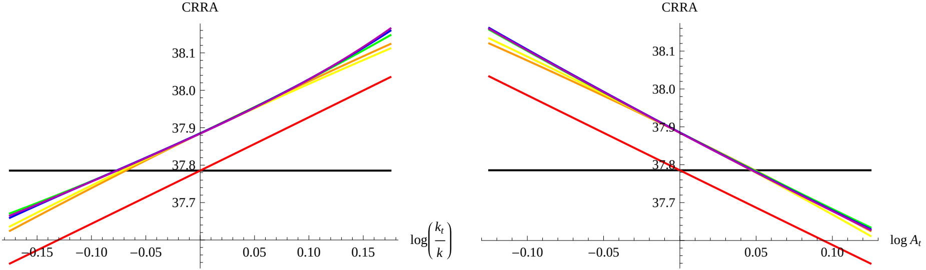


Dashed black line denotes closed-form value of .9143.
(Compare to $\gamma = 2$)

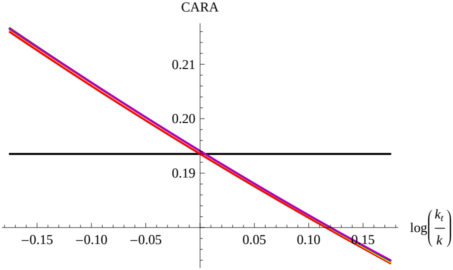
Risk Aversion Away from the Steady State

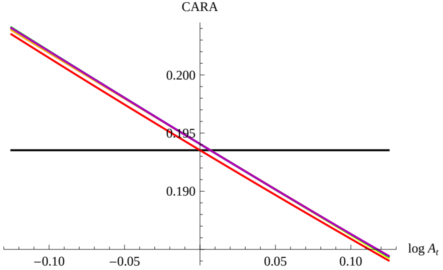
Same exercise with Epstein-Zin Preferences (higher risk aversion):



Risk Aversion Away from the Steady State

Absolute risk aversion is countercyclical:





Asset Pricing

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Risk premium on the asset:

$$\frac{E_t m_{t+1} E_t p_{t+1} - E_t m_{t+1} p_{t+1}}{E_t m_{t+1} E_t p_{t+1}}$$

$$= \frac{-\text{Cov}_t(m_{t+1}, p_{t+1})}{E_t m_{t+1} E_t p_{t+1}}$$

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Asset Pricing

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$$dl_{t+1}^* = -\lambda dc_{t+1}^*,$$

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$$\frac{-\text{Cov}_t(dm_{t+1}, dp_{t+1})}{\beta} = R^a(a; \theta) \cdot \text{Cov}_t(da_{t+1}, dp_{t+1}).$$

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$$+ \frac{u_1 u_{12}}{u_{11} u_{22} - u_{12}^2} dw_{t+1}$$
$$+ \frac{-u_1}{u_{11} - \lambda u_{12}} E_{t+1} \sum_{k=1}^{\infty} \frac{1}{(1+r)^k} \left(\frac{r\lambda}{1+w\lambda} dw_{t+k} - \beta dr_{t+k+1} \right).$$

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Risk premium on the asset:

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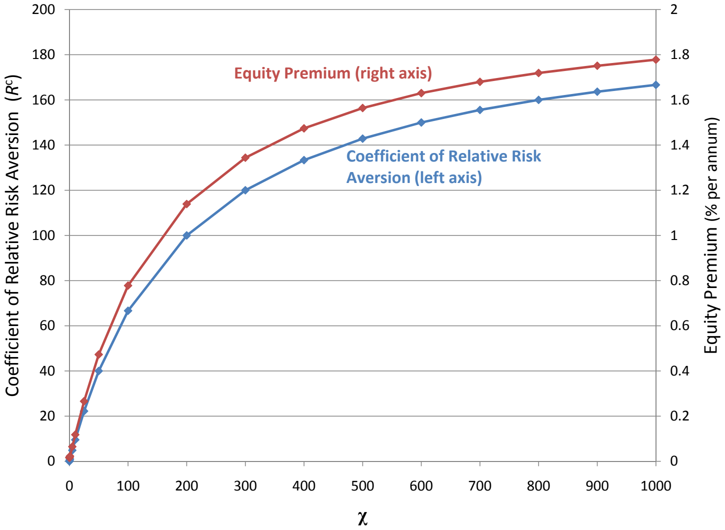
Asset Pricing

Numerical results for an equity claim to consumption, $\gamma = 200$:



Risk Neutrality

Hansen-Rogerson linear-labor preferences are common:

- Extensive labor margin: Hansen (1985), Rogerson (1988)
- Monetary search: Lagos-Wright (2005)
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The present paper suggests ways to model risk neutrality that do not require linear utility of consumption.

Empirical Estimates of Risk Aversion

Barsky-Juster-Kimball-Shaprio (1997):

> "Suppose that you are the only income earner in the family, and you have a good job guaranteed to give you your current (family) income every year for life. You are given the opportunity to take a new and equally good job, with a 50–50 chance it will double your (family) income and a 50–50 chance that it will cut your (family) income by a third. Would you take the new job?"

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Empirical estimates of risk aversion using methods like these remain valid in the framework of the present paper.

What is different is how these estimates are mapped into model parameters (i.e., risk aversion $\neq -cu_{11}/u_1$)

Empirical Asset Pricing

Campbell (1996, 1999): $u(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma}, \quad m_{t+1} = \log \beta - \gamma \Delta c_{t+1}$

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Country	$E_t(r_{e,t} - r_{f,t})$	std$(r_{e,t} - r_{f,t})$	std(Δc)	γ
USA	5.82	17.0	0.91	37.3
JPN	6.83	21.6	2.35	13.4
GER	6.77	20.4	2.50	13.3
FRA	7.12	22.8	2.13	14.6
UK	8.31	21.6	2.59	14.9
ITA	2.17	27.3	1.68	4.7
CAN	3.04	16.7	2.03	9.0

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If $u(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma} - \eta\frac{L_t^{1+\chi}}{1+\chi}$, then $\gamma \neq$ risk aversion.

Conclusions

1. The labor margin has dramatic effects on risk aversion
2. Risk aversion is the right concept for asset pricing
3. Arrow-Pratt risk neutrality holds for any u with $u_{11}u_{22} - u_{12}^2 = 0$
4. Risk aversion and the intertemporal elasticity of substitution are nonreciprocal when there is labor in the model
5. Simple, closed-form expressions for risk aversion in DSGE models with:
   - expected utility preferences
   - Epstein-Zin preferences
   - external or internal habits
   - valid away from steady state