Implications of Labor Market Frictions for Risk Aversion and Risk Premia

Eric T. Swanson
University of California, Irvine

Macroeconomics Seminar
Bilkent University
December 14, 2018

Coefficient of Relative Risk Aversion

Suppose a household has preferences:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t),$$

$$u(c_t, l_t) = \frac{c_t^{1-\gamma}}{1-\gamma} - \eta\, l_t$$

Coefficient of Relative Risk Aversion

Suppose a household has preferences:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t),$$

$$u(c_t, l_t) = \frac{c_t^{1-\gamma}}{1-\gamma} - \eta\, l_t$$

What is the household's coefficient of relative risk aversion?

Coefficient of Relative Risk Aversion

Suppose a household has preferences:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t),$$

$$u(c_t, l_t) = \frac{c_t^{1-\gamma}}{1-\gamma} - \eta\, l_t$$

What is the household's coefficient of relative risk aversion?

Answer: 0

Coefficient of Relative Risk Aversion

Suppose a household has preferences:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t),$$

$$u(c_t, l_t) = \frac{c_t^{1-\gamma}}{1-\gamma} - \eta \frac{l_t^{1+\chi}}{1+\chi}$$

What is the household's coefficient of relative risk aversion?

Answer: $\dfrac{1}{\frac{1}{\gamma} + \frac{1}{\chi}}$

Empirical Relevance of the Labor Margin

Imbens, Rubin, and Sacerdote (2001):

- Lottery winners reduce labor supply by \$.11 per \$1 of prize

Empirical Relevance of the Labor Margin

Imbens, Rubin, and Sacerdote (2001):

- Lottery winners reduce labor supply by \$.11 per \$1 of prize

Cesarini, Lindqvist, Notowidigdo, and Östling (2017):

- Swedish lottery winners reduce labor supply by SEK .11/SEK won
- Spouses also reduce labor supply (but by less)
- Labor response is primarily due to reduction in *hours*

Empirical Relevance of the Labor Margin

Imbens, Rubin, and Sacerdote (2001):

- Lottery winners reduce labor supply by $.11 per $1 of prize

Cesarini, Lindqvist, Notowidigdo, and Östling (2017):

- Swedish lottery winners reduce labor supply by SEK .11/SEK won
- Spouses also reduce labor supply (but by less)
- Labor response is primarily due to reduction in *hours*

Coile and Levine (2009):

- Older individuals are 7% less likely to retire in a given year after a 30% fall in stock market

Empirical Relevance of the Labor Margin

Imbens, Rubin, and Sacerdote (2001):

- Lottery winners reduce labor supply by \$.11 per \$1 of prize

Cesarini, Lindqvist, Notowidigdo, and Östling (2017):

- Swedish lottery winners reduce labor supply by SEK .11/SEK won
- Spouses also reduce labor supply (but by less)
- Labor response is primarily due to reduction in *hours*

Coile and Levine (2009):

- Older individuals are 7% less likely to retire in a given year after a 30% fall in stock market

Coronado and Perozek (2003):

- Individuals who held more stocks in late 1990s retired 7 months earlier

Empirical Relevance of the Labor Margin

Imbens, Rubin, and Sacerdote (2001):

- Lottery winners reduce labor supply by $.11 per $1 of prize

Cesarini, Lindqvist, Notowidigdo, and Östling (2017):

- Swedish lottery winners reduce labor supply by SEK .11/SEK won
- Spouses also reduce labor supply (but by less)
- Labor response is primarily due to reduction in *hours*

Coile and Levine (2009):

- Older individuals are 7% less likely to retire in a given year after a 30% fall in stock market

Coronado and Perozek (2003):

- Individuals who held more stocks in late 1990s retired 7 months earlier

Large literature finds significantly negative wealth effect on labor supply (e.g., Pencavel 1986)

Frictional Labor Markets

No labor/perfectly rigid labor market:

- Arrow (1964), Pratt (1965)
- Epstein-Zin (1989), Weil (1990)

Frictional Labor Markets

No labor/perfectly rigid labor market:

- Arrow (1964), Pratt (1965)
- Epstein-Zin (1989), Weil (1990)

Perfectly flexible labor market:

- Swanson (2012, 2018)

Frictional Labor Markets

No labor/perfectly rigid labor market:

- Arrow (1964), Pratt (1965)
- Epstein-Zin (1989), Weil (1990)

Perfectly flexible labor market:

- Swanson (2012, 2018)

Frictional labor market:

- this paper

A Household

Household preferences:

$$E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[U(c_\tau) - V(l_\tau + u_\tau) \right],$$

A Household

Household preferences:

$$E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[U(c_\tau) - V(l_\tau + u_\tau) \right],$$

Flow budget constraint:

$$a_{\tau+1} = (1 + r_\tau) a_\tau + w_\tau l_\tau + d_\tau - c_\tau,$$

A Household

Household preferences:

$$E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[U(c_\tau) - V(l_\tau + u_\tau) \right],$$

Flow budget constraint:

$$a_{\tau+1} = (1 + r_\tau) a_\tau + w_\tau l_\tau + d_\tau - c_\tau,$$

No-Ponzi condition:

$$\lim_{T\to\infty} \prod_{\tau=t}^{T} (1 + r_{\tau+1})^{-1} a_{T+1} \geq 0,$$

A Household

Household preferences:

$$E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[U(c_\tau) - V(l_\tau + u_\tau) \right],$$

Flow budget constraint:

$$a_{\tau+1} = (1 + r_\tau) a_\tau + w_\tau l_\tau + d_\tau - c_\tau,$$

No-Ponzi condition:

$$\lim_{T \to \infty} \prod_{\tau=t}^{T} (1 + r_{\tau+1})^{-1} a_{T+1} \geq 0,$$

$\{w_\tau, r_\tau, d_\tau\}$ are exogenous processes, governed by Θ_τ

A Household

Household preferences:

$$E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[U(c_\tau) - V(l_\tau + u_\tau) \right],$$

Flow budget constraint:

$$a_{\tau+1} = (1 + r_\tau) a_\tau + w_\tau l_\tau + d_\tau - c_\tau,$$

No-Ponzi condition:

$$\lim_{T \to \infty} \prod_{\tau=t}^{T} (1 + r_{\tau+1})^{-1} a_{T+1} \geq 0,$$

$\{w_\tau, r_\tau, d_\tau\}$ are exogenous processes, governed by Θ_τ

Labor market search: $l_{\tau+1} = (1-s) l_\tau + f(\Theta_\tau) u_\tau$

The Value Function

State variables of the household's problem are $(a_t, l_t; \Theta_t)$.

Let:

$$c_t^* \equiv c^*(a_t, l_t; \Theta_t),$$

$$u_t^* \equiv u^*(a_t, l_t; \Theta_t).$$

The Value Function

State variables of the household's problem are $(a_t, l_t; \Theta_t)$.

Let:

$$c_t^* \equiv c^*(a_t, l_t; \Theta_t),$$
$$u_t^* \equiv u^*(a_t, l_t; \Theta_t).$$

Value function, Bellman equation:

$$\mathbb{V}(a_t, l_t; \Theta_t) = U(c_t^*) - V(l_t + u_t^*) + \beta E_t \mathbb{V}(a_{t+1}^*, l_{t+1}^*; \Theta_{t+1}),$$

where:

$$a_{t+1}^* \equiv (1 + r_t)a_t + w_t l_t + d_t - c_t^*,$$
$$l_{t+1}^* \equiv (1-s)l_t + f(\Theta_t)u_t^*.$$

Technical Conditions

Assumption 1. *The function $U(c_t)$ is increasing, twice-differentiable, and strictly concave, and $V(l_t)$ is increasing, twice-differentiable, and strictly convex.*

Assumption 2. *A solution $\mathbb{V} : X \to \mathbb{R}$ to the household's generalized Bellman equation exists and is unique, continuous, and concave.*

Assumption 3. *For any $(a_t, l_t; \Theta_t) \in X$, the household's optimal choice (c_t^*, u_t^*) exists, is unique, and lies in the interior of $\Gamma(a_t, l_t; \Theta_t)$.*

Assumption 4. *For any $(a_t, l_t; \Theta_t)$ in the interior of X, the second derivatives of $\mathbb{V}$ with respect to its first two arguments, $\mathbb{V}_{11}(a_t, l_t; \Theta_t)$, $\mathbb{V}_{12}(a_t, l_t; \Theta_t)$, and $\mathbb{V}_{22}(a_t, l_t; \Theta_t)$, exist.*

Assumptions about the Economic Environment

Assumption 5. *The household is* infinitesimal*.*

Assumption 6. *The household is* representative*.*

Assumption 7. *The model has a* nonstochastic steady state*,* $x_t = x_{t+k}$ *for* $k = 1, 2, \ldots,$ *and* $x \in \{c, u, l, a, w, r, d, \Theta\}$*.*

Assumptions about the Economic Environment

Assumption 5. *The household is* infinitesimal*.*

Assumption 6. *The household is* representative*.*

Assumption 7. *The model has a* nonstochastic steady state*,* $x_t = x_{t+k}$ *for* $k = 1, 2, \ldots,$ *and* $x \in \{c, u, l, a, w, r, d, \Theta\}$*.*

Assumption 7′. *The model has a* balanced growth path *that can be renormalized to a nonstochastic steady state after a suitable change of variables.*

Arrow-Pratt in a Static One-Good Model (Review)

Compare:

$$E\,u(c + \sigma\varepsilon) \quad \text{vs.} \quad u(c - \mu)$$

Arrow-Pratt in a Static One-Good Model (Review)

Compare:

$$E\,u(c+\sigma\varepsilon) \quad \text{vs.} \quad u(c-\mu)$$

Compute:

$$u(c-\mu) \;\approx\; u(c) - \mu u'(c),$$

Arrow-Pratt in a Static One-Good Model (Review)

Compare:

$$E\,u(c+\sigma\varepsilon) \quad \text{vs.} \quad u(c-\mu)$$

Compute:

$$u(c-\mu) \;\approx\; u(c) - \mu u'(c),$$

$$E\,u(c+\sigma\varepsilon) \;\approx\; u(c) + u'(c)\sigma E[\varepsilon] + \frac{1}{2}u''(c)\sigma^2 E[\varepsilon^2],$$

Arrow-Pratt in a Static One-Good Model (Review)

Compare:

$$E\,u(c+\sigma\varepsilon) \quad \text{vs.} \quad u(c-\mu)$$

Compute:

$$u(c-\mu) \approx u(c) - \mu u'(c),$$

$$E\,u(c+\sigma\varepsilon) \approx u(c) + \frac{1}{2}u''(c)\sigma^2.$$

Arrow-Pratt in a Static One-Good Model (Review)

Compare:

$$E\, u(c + \sigma\varepsilon) \quad \text{vs.} \quad u(c - \mu)$$

Compute:

$$u(c-\mu) \approx u(c) - \mu u'(c),$$

$$E\, u(c + \sigma\varepsilon) \approx u(c) + \frac{1}{2} u''(c)\sigma^2.$$

$$\mu = \frac{-u''(c)}{u'(c)} \frac{\sigma^2}{2}.$$

Arrow-Pratt in a Static One-Good Model (Review)

Compare:

$$E\,u(c+\sigma\varepsilon) \quad \text{vs.} \quad u(c-\mu)$$

Compute:

$$u(c-\mu) \approx u(c) - \mu u'(c),$$

$$E\,u(c+\sigma\varepsilon) \approx u(c) + \frac{1}{2}u''(c)\sigma^2.$$

$$\mu = \frac{-u''(c)}{u'(c)}\,\frac{\sigma^2}{2}.$$

Coefficient of absolute risk aversion is defined to be:

$$\lim_{\sigma\to 0} 2\mu(\sigma)/\sigma^2 = \frac{-u''(c)}{u'(c)}.$$

Arrow-Pratt in a Dynamic Model

Arrow-Pratt in a Dynamic Model

Consider a one-shot gamble in period t:

$$a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t + \sigma\varepsilon_{t+1}, \qquad (*)$$

Arrow-Pratt in a Dynamic Model

Consider a one-shot gamble in period t:

$$a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t + \sigma\varepsilon_{t+1}, \qquad (*)$$

Note we cannot easily consider gambles over:

- a_t (state variable, already known at t)
- c_t (choice variable)

Arrow-Pratt in a Dynamic Model

Consider a one-shot gamble in period t:

$$a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t + \sigma\varepsilon_{t+1}, \qquad (*)$$

Note we cannot easily consider gambles over:

- a_t (state variable, already known at t)
- c_t (choice variable)

Note $(*)$ is equivalent to gamble over asset returns:

$$a_{t+1} = (1 + r_t + \sigma\tilde{\varepsilon}_{t+1})a_t + w_t l_t + d_t - c_t.$$

or income:

$$a_{t+1} = (1 + r_t)a_t + w_t l_t + (d_t + \sigma\varepsilon_{t+1}) - c_t,$$

Arrow-Pratt in a Dynamic Model

Consider a one-shot gamble in period t:

$$a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t + \sigma\varepsilon_{t+1},$$

vs.

$$a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t - \mu.$$

Arrow-Pratt in a Dynamic Model

Consider a one-shot gamble in period t:

$$a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t + \sigma\varepsilon_{t+1},$$

vs.

$$a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t - \mu.$$

Welfare loss from μ:

$$\mathbb{V}_1(a_t, l_t; \Theta_t)\, \frac{\mu}{(1 + r_t)}$$

Arrow-Pratt in a Dynamic Model

Consider a one-shot gamble in period t:

$$a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t + \sigma\varepsilon_{t+1},$$

vs.

$$a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t - \mu.$$

Welfare loss from μ:

$$\beta E_t \mathbb{V}_1(a^*_{t+1}, l^*_{t+1}; \Theta_{t+1})\,\mu.$$

Arrow-Pratt in a Dynamic Model

Consider a one-shot gamble in period t:

$$a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t + \sigma\varepsilon_{t+1},$$

vs.

$$a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t - \mu.$$

Welfare loss from μ:

$$\beta E_t \mathbb{V}_1(a^*_{t+1}, l^*_{t+1}; \Theta_{t+1})\, \mu.$$

Loss from σ:

$$\beta E_t \mathbb{V}_{11}(a^*_{t+1}, l^*_{t+1}; \Theta_{t+1}) \frac{\sigma^2}{2}.$$

Coefficient of Absolute Risk Aversion

Definition 1. *The household's coefficient of absolute risk aversion at* $(a_t, l_t; \Theta_t)$ *is given by* $R^a(a_t, l_t; \Theta_t) = \lim_{\sigma \to 0} 2\mu(\sigma)/\sigma^2$.

Coefficient of Absolute Risk Aversion

Definition 1. *The household's coefficient of absolute risk aversion at* $(a_t, l_t; \Theta_t)$ *is given by* $R^a(a_t, l_t; \Theta_t) = \lim_{\sigma \to 0} 2\mu(\sigma)/\sigma^2$.

Proposition 1. *The household's coefficient of absolute risk aversion at* $(a_t, l_t; \Theta_t)$ *is well-defined and satisfies*

$$R^a(a_t, l_t; \Theta_t) \;=\; \frac{-E_t \mathbb{V}_{11}(a^*_{t+1}, l^*_{t+1}; \Theta_{t+1})}{E_t \mathbb{V}_1(a^*_{t+1}, l^*_{t+1}; \Theta_{t+1})}\,.$$

Coefficient of Absolute Risk Aversion

Definition 1. *The household's coefficient of absolute risk aversion at* $(a_t, l_t; \Theta_t)$ *is given by* $R^a(a_t, l_t; \Theta_t) = \lim_{\sigma \to 0} 2\mu(\sigma)/\sigma^2$.

Proposition 1. *The household's coefficient of absolute risk aversion at* $(a_t, l_t; \Theta_t)$ *is well-defined and satisfies*

$$R^a(a_t, l_t; \Theta_t) \;=\; \frac{-E_t \mathbb{V}_{11}(a^*_{t+1}, l^*_{t+1}; \Theta_{t+1})}{E_t \mathbb{V}_1(a^*_{t+1}, l^*_{t+1}; \Theta_{t+1})}\,.$$

Folk wisdom: Constantinides (1990), Farmer (1990), Campbell-Cochrane (1999), Boldrin-Christiano-Fisher (1997, 2001), Flavin-Nakagawa (2008)

Coefficient of Absolute Risk Aversion

Definition 1. *The household's coefficient of absolute risk aversion at* $(a_t, l_t; \Theta_t)$ *is given by* $R^a(a_t, l_t; \Theta_t) = \lim_{\sigma \to 0} 2\mu(\sigma)/\sigma^2$.

Proposition 1. *The household's coefficient of absolute risk aversion at* $(a_t, l_t; \Theta_t)$ *is well-defined and satisfies*

$$R^a(a_t, l_t; \Theta_t) = \frac{-E_t \mathbb{V}_{11}(a^*_{t+1}, l^*_{t+1}; \Theta_{t+1})}{E_t \mathbb{V}_1(a^*_{t+1}, l^*_{t+1}; \Theta_{t+1})} .$$

Evaluated at the nonstochastic steady state, this simplifies to:

$$R^a(a, l; \Theta) = \frac{-\mathbb{V}_{11}(a, l; \Theta)}{\mathbb{V}_1(a, l; \Theta)}$$

Folk wisdom: Constantinides (1990), Farmer (1990), Campbell-Cochrane (1999), Boldrin-Christiano-Fisher (1997, 2001), Flavin-Nakagawa (2008)

Solve for $\mathbb{V}_1$ and $\mathbb{V}_{11}$

Household preferences:

$$E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[U(c_\tau) - V(l_\tau + u_\tau) \right]$$

Solve for $\mathbb{V}_1$ and $\mathbb{V}_{11}$

Household preferences:

$$E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[U(c_\tau) - V(l_\tau + u_\tau)\right]$$

Benveniste-Scheinkman:

$$\mathbb{V}_1(a_t, l_t; \Theta_t) = (1 + r_t)\, U'(c_t^*). \qquad (*)$$

Solve for $\mathbb{V}_1$ and $\mathbb{V}_{11}$

Household preferences:

$$E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[U(c_\tau) - V(l_\tau + u_\tau)\right]$$

Benveniste-Scheinkman:

$$\mathbb{V}_1(a_t, l_t; \Theta_t) = (1 + r_t)\, U'(c_t^*). \qquad (*)$$

Differentiate $(*)$ to get:

$$\mathbb{V}_{11}(a_t, l_t; \Theta_t) \; = \; (1 + r_t) U''(c_t^*) \frac{\partial c_t^*}{\partial a_t} .$$

Solve for $\partial c_t^* / \partial a_t$

Solve for $\partial c_t^* / \partial a_t$

Consumption Euler equation:

$$U'(c_t^*) \;=\; \beta E_t (1 + r_{t+1})\, U'(c_{t+1}^*),$$

Solve for $\partial c_t^* / \partial a_t$

Consumption Euler equation:

$$U'(c_t^*) \; = \; \beta E_t (1 + r_{t+1}) \, U'(c_{t+1}^*),$$

implies, at steady state:

$$\frac{\partial c_t^*}{\partial a_t} \; = \; E_t \frac{\partial c_{t+1}^*}{\partial a_t} \; = \; E_t \frac{\partial c_{t+k}^*}{\partial a_t} \, , \quad k = 1, 2, \ldots$$

Solve for $\partial c_t^* / \partial a_t$

Consumption Euler equation:

$$U'(c_t^*) \;=\; \beta E_t (1 + r_{t+1})\, U'(c_{t+1}^*),$$

implies, at steady state:

$$\frac{\partial c_t^*}{\partial a_t} \;=\; E_t \frac{\partial c_{t+1}^*}{\partial a_t} \;=\; E_t \frac{\partial c_{t+k}^*}{\partial a_t}\,, \quad k = 1, 2, \ldots$$

Household's budget constraint, no-Ponzi condition imply:

$$\sum_{k=0}^{\infty} \frac{1}{(1+r)^k}\, E_t \left[\frac{\partial c_{t+k}^*}{\partial a_t} - w \frac{\partial l_{t+k}^*}{\partial a_t} \right] \;=\; 1 + r\,.$$

Solve for $\partial l^*_{t+k}/\partial a_t$

Labor search (unemployment) Euler equation:

$$\frac{V'(l_t + u_t^*)}{f(\Theta_t)} = \beta E_t\Big[w_{t+1}U'(c^*_{t+1}) - V'(l^*_{t+1} + u^*_{t+1}) + (1-s)\frac{V'(l^*_{t+1} + u^*_{t+1})}{f(\Theta_{t+1})}\Big]$$

Solve for $\partial l^*_{t+k} / \partial a_t$

Labor search (unemployment) Euler equation:

$$\frac{V'(l_t + u^*_t)}{f(\Theta_t)} = \beta E_t \Big[w_{t+1} U'(c^*_{t+1}) - V'(l^*_{t+1} + u^*_{t+1}) + (1-s) \frac{V'(l^*_{t+1} + u^*_{t+1})}{f(\Theta_{t+1})} \Big]$$

and transition equation

$$l_{t+1} = (1-s) l_t + f(\Theta_t) u_t$$

Solve for $\partial l^*_{t+k}/\partial a_t$

Labor search (unemployment) Euler equation:

$$\frac{V'(l_t + u_t^*)}{f(\Theta_t)} = \beta E_t\Big[w_{t+1}U'(c^*_{t+1}) - V'(l^*_{t+1}+u^*_{t+1}) + (1-s)\frac{V'(l^*_{t+1}+u^*_{t+1})}{f(\Theta_{t+1})}\Big]$$

and transition equation

$$l_{t+1} = (1-s)l_t + f(\Theta_t)u_t$$

imply, at steady state:

$$E_t\frac{\partial l^*_{t+k}}{\partial a_t} = -\frac{\gamma}{\chi}\,\frac{l+u}{c}\,\frac{f(\Theta)}{s+f(\Theta)}\left[1-\left(1-s-f(\Theta)\right)^k\right]\frac{\partial c^*_t}{\partial a_t}.$$

where $\gamma \equiv -cU''(c)/U'(c)$, $\chi \equiv (l+u)V''(l+u)/V'(l+u)$

Solve for $\partial c_t^* / \partial a_t$

Household's budget constraint, no-Ponzi condition:

$$\sum_{k=0}^{\infty} \frac{1}{(1+r)^k} E_t \left[\frac{\partial c_{t+k}^*}{\partial a_t} - w \frac{\partial l_{t+k}^*}{\partial a_t} \right] = 1 + r \,,$$

Solve for $\partial c_t^* / \partial a_t$

Household's budget constraint, no-Ponzi condition:

$$\sum_{k=0}^{\infty} \frac{1}{(1+r)^k} E_t \left[\frac{\partial c_{t+k}^*}{\partial a_t} - w \frac{\partial l_{t+k}^*}{\partial a_t} \right] = 1 + r \, ,$$

Consumption Euler equation:

$$\frac{\partial c_t^*}{\partial a_t} = E_t \frac{\partial c_{t+1}^*}{\partial a_t} = E_t \frac{\partial c_{t+k}^*}{\partial a_t} \, , \quad k = 1, 2, \ldots ,$$

Solve for $\partial c_t^* / \partial a_t$

Household's budget constraint, no-Ponzi condition:

$$\sum_{k=0}^{\infty} \frac{1}{(1+r)^k} E_t \left[\frac{\partial c_{t+k}^*}{\partial a_t} - w \frac{\partial l_{t+k}^*}{\partial a_t} \right] = 1 + r \,,$$

Consumption Euler equation:

$$\frac{\partial c_t^*}{\partial a_t} = E_t \frac{\partial c_{t+1}^*}{\partial a_t} = E_t \frac{\partial c_{t+k}^*}{\partial a_t} \,, \quad k = 1, 2, \ldots,$$

Labor Euler equation:

$$E_t \frac{\partial l_{t+k}^*}{\partial a_t} = -\frac{\gamma}{\chi} \frac{l+u}{c} \frac{f(\Theta)}{s + f(\Theta)} \left[1 - (1 - s - f(\Theta))^k \right] \frac{\partial c_t^*}{\partial a_t} \,,$$

Solve for $\partial c_t^* / \partial a_t$

Household's budget constraint, no-Ponzi condition:

$$\sum_{k=0}^{\infty} \frac{1}{(1+r)^k} E_t \left[\frac{\partial c_{t+k}^*}{\partial a_t} - w \frac{\partial l_{t+k}^*}{\partial a_t} \right] = 1 + r \,,$$

Consumption Euler equation:

$$\frac{\partial c_t^*}{\partial a_t} = E_t \frac{\partial c_{t+1}^*}{\partial a_t} = E_t \frac{\partial c_{t+k}^*}{\partial a_t} \,, \quad k = 1, 2, \ldots ,$$

Labor Euler equation:

$$E_t \frac{\partial l_{t+k}^*}{\partial a_t} = -\frac{\gamma}{\chi} \frac{l+u}{c} \frac{f(\Theta)}{s + f(\Theta)} \left[1 - \left(1 - s - f(\Theta)\right)^k \right] \frac{\partial c_t^*}{\partial a_t} \,,$$

Solution is a "modified permanent income hypothesis":

$$\frac{\partial c_t^*}{\partial a_t} = \frac{r}{1 + \frac{\gamma}{\chi} \frac{w(l+u)}{c} \frac{f(\Theta)}{r + s + f(\Theta)}} \,.$$

Solve for Coefficient of Absolute Risk Aversion

$$\mathbb{V}_1(a, l; \theta) = (1 + r)\, U'(c),$$

Solve for Coefficient of Absolute Risk Aversion

$$\mathbb{V}_1(a, l; \theta) = (1 + r)\, U'(c),$$

$$\mathbb{V}_{11}(a, l; \theta) \; = \; (1 + r) U''(c)\, \frac{\partial c_t^*}{\partial a_t},$$

Solve for Coefficient of Absolute Risk Aversion

$$\mathbb{V}_1(a,l;\theta) = (1+r)\,U'(c),$$

$$\mathbb{V}_{11}(a,l;\theta) \;=\; (1+r)U''(c)\,\frac{\partial c_t^*}{\partial a_t},$$

$$\frac{\partial c_t^*}{\partial a_t} \;=\; \frac{r}{1+\dfrac{\gamma}{\chi}\,\dfrac{w(l+u)}{c}\,\dfrac{f(\Theta)}{r+s+f(\Theta)}},$$

Solve for Coefficient of Absolute Risk Aversion

$$\mathbb{V}_1(a, l; \theta) = (1 + r)\, U'(c),$$

$$\mathbb{V}_{11}(a, l; \theta) \;=\; (1 + r) U''(c)\, \frac{\partial c_t^*}{\partial a_t},$$

$$\frac{\partial c_t^*}{\partial a_t} \;=\; \frac{r}{1 + \frac{\gamma}{\chi}\, \frac{w(l+u)}{c}\, \frac{f(\Theta)}{r + s + f(\Theta)}},$$

Proposition 2. *Given Assumptions 1–7, the household's coefficient of absolute risk aversion, $R^a(a_t, l_t; \Theta_t)$, evaluated at steady state, satisfies*

$$R^a(a, l; \Theta) \;=\; \frac{-U''(c)}{U'(c)}\, \frac{r}{1 + \frac{\gamma}{\chi}\, \frac{w(l+u)}{c}\, \frac{f(\Theta)}{r + s + f(\Theta)}}.$$

Relative Risk Aversion

Compare: $a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t + \sigma A_t \varepsilon_{t+1}$

vs.

$$a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t - \mu A_t.$$

Relative Risk Aversion

Compare: $a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t + \sigma A_t \varepsilon_{t+1}$

vs.

$$a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t - \mu A_t.$$

Definition 2. *The households' coefficient of relative risk aversion, $R^c(a_t, l_t; \Theta_t) \equiv A_t R^a(a_t, l_t; \Theta_t)$, where A_t denotes the household's financial assets plus present discounted value of labor income.*

At steady state, $A = c/r$, and

$$R^c(a; \Theta) = \frac{-U''(c)}{U'(c)} \frac{c}{1 + \frac{\gamma}{\chi} \frac{w(l+u)}{c} \frac{f(\Theta)}{r + s + f(\Theta)}}.$$

Numerical Example

Household period utility function:

$$\frac{c_t^{1-\gamma}}{1-\gamma} - \chi_0 \frac{(l_t + u_t)^{1+\chi}}{1+\chi}$$

Numerical Example

Household period utility function:

$$\frac{c_t^{1-\gamma}}{1-\gamma} - \chi_0 \frac{(l_t + u_t)^{1+\chi}}{1+\chi}$$

Economy is a simple RBC model with labor market frictions:

- Competitive firms,
- Cobb-Douglas production functions, $y_t = Z_t k_t^{1-\phi} l_t^{\phi}$
- AR(1) technology, $\log Z_{t+1} = \rho_z \log Z_t + \varepsilon_t$
- Capital accumulation, $k_{t+1} = (1-\delta)k_t + y_t - c_t$
- Labor market frictions, $l_{t+1} = (1-s)l_t + h_t$

Numerical Example

Labor market search:

- Cobb-Douglas matching function, $h_t = \mu u_t^{1-\eta} v_t^{\eta}$
- Wage set by Nash bargaining with equal weights

Numerical Example

Labor market search:

- Cobb-Douglas matching function, $h_t = \mu u_t^{1-\eta} v_t^{\eta}$
- Wage set by Nash bargaining with equal weights

Equity security:

- Equity is a consumption claim
- Equity premium is expected excess return,

$$\psi_t \equiv \frac{E_t(C_{t+1} + p_{t+1})}{p_t} - (1 + r_t^f)$$

Numerical Example

Labor market search:

- Cobb-Douglas matching function, $h_t = \mu u_t^{1-\eta} v_t^{\eta}$
- Wage set by Nash bargaining with equal weights

Equity security:

- Equity is a consumption claim
- Equity premium is expected excess return,

$$\psi_t \equiv \frac{E_t(C_{t+1} + p_{t+1})}{p_t} - (1 + r_t^f)$$

Baseline calibration:

- Production: $\phi = 0.7$, $\delta = .0083$, $\rho_z = 0.99$, $\sigma_\varepsilon = .005$
- Matching: $s = .02$, $\eta = 0.5$, $v/u = 0.6$, $f(\Theta) = 0.28$
- Preferences: $\beta = .996$, $\gamma = 100$, $\chi = 100$, $l + u = 0.3$

Figure 1: Risk Aversion and Equity Premium vs. χ

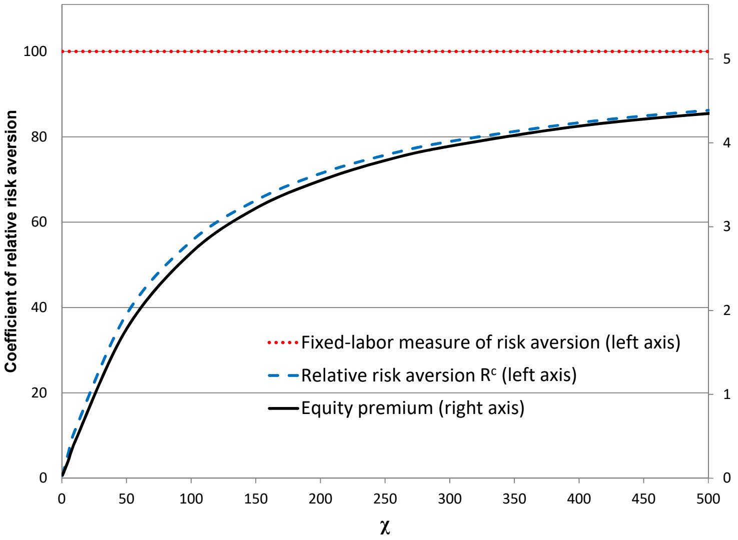


Figure 2: Risk Aversion and Equity Premium vs. γ

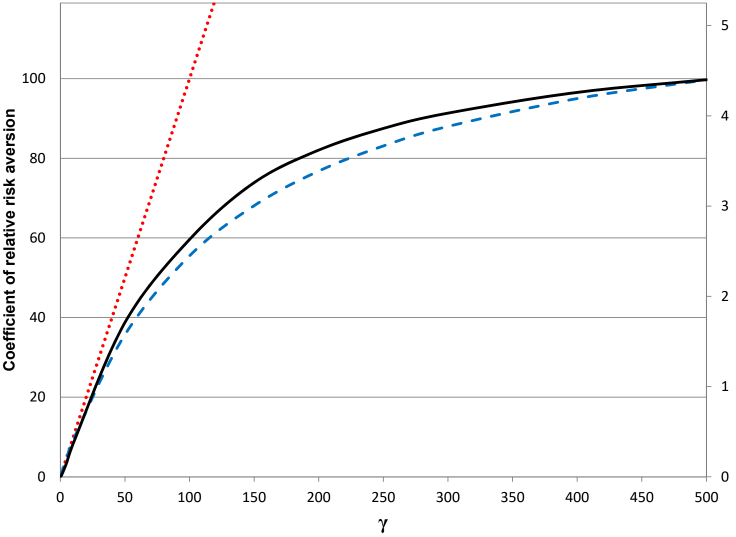
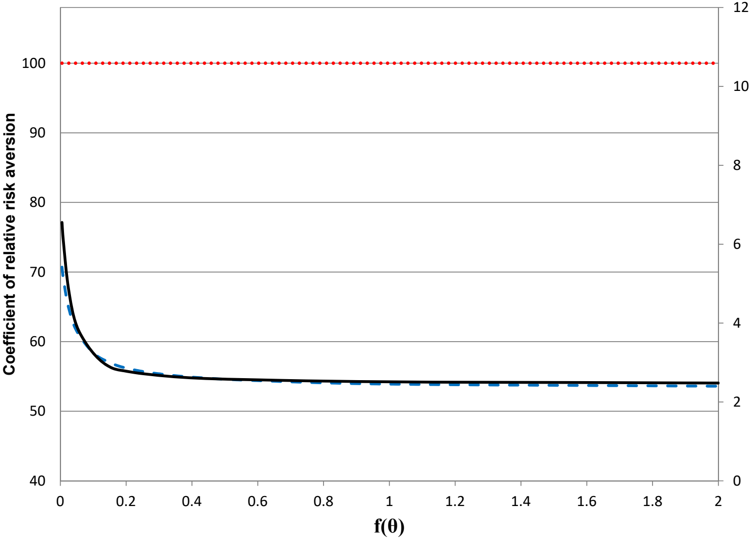


Figure 3: Risk Aversion and Equity Premium vs. $f(\Theta)$



Risk Aversion Higher in More Frictional Labor Markets

Proposition 3. *Let $f_1, f_2 : \Omega_\Theta \to [0, 1]$. Given Assumptions 1–8 and fixed values for the parameters s, β, γ, and χ, let $(a_1, l_1; \Theta_1)$ and $(a_2, l_2; \Theta_2)$ denote corresponding steady-state values of $(a_t, l_t; \Theta_t)$. If $f_1(\Theta_1) < f_2(\Theta_2)$, then $R_1^c(a_1, l_1; \Theta_1) > R_2^c(a_2, l_2; \Theta_2)$.*

Risk Aversion Higher in More Frictional Labor Markets

Proposition 3. *Let $f_1, f_2 : \Omega_\Theta \to [0,1]$. Given Assumptions 1–8 and fixed values for the parameters s, β, γ, and χ, let $(a_1, l_1; \Theta_1)$ and $(a_2, l_2; \Theta_2)$ denote corresponding steady-state values of $(a_t, l_t; \Theta_t)$. If $f_1(\Theta_1) < f_2(\Theta_2)$, then $R_1^c(a_1, l_1; \Theta_1) > R_2^c(a_2, l_2; \Theta_2)$.*

Proof:

$$R^c(a, l; \Theta) = \frac{-U''(c)}{U'(c)} \frac{c}{1 + \frac{\gamma}{\chi} \frac{wl}{c} \frac{s + f(\Theta)}{r + s + f(\Theta)}}$$

is decreasing in $f(\Theta)$.

Risk Aversion Is Higher in Recessions

Proposition 4. *Given Assumptions 1–8 and fixed values for the parameters s,* β, γ, *and* χ, $R^c(a, l; \Theta)$ *is decreasing in* l/u.

Risk Aversion Is Higher in Recessions

Proposition 4. *Given Assumptions 1–8 and fixed values for the parameters s, β, γ, and χ, $R^c(a, l; \Theta)$ is decreasing in l/u.*

Proof:

$$R^c(a,l;\Theta) = \frac{-U''(c)}{U'(c)} \frac{c}{1 + \frac{\gamma}{\chi} \frac{w(l+u)}{c} \frac{f(\Theta)}{r+s+f(\Theta)}}.$$

Using $sl = f(\Theta)u$,

$$R^c(a,l;\Theta) = \frac{-U''(c)}{U'(c)} \frac{c}{1 + \frac{\gamma}{\chi} \frac{wl}{c} \frac{s(1+l/u)}{r+s(1+l/u)}}.$$

Risk Aversion Higher for Less Employable Households

Two types of households:

- Measure 1 of type 1 households
- Measure 0 ot type 2 households
- Type 1 households are more employable: $f_1(\Theta) > f_2(\Theta)$

Risk Aversion Higher for Less Employable Households

Two types of households:

- Measure 1 of type 1 households
- Measure 0 ot type 2 households
- Type 1 households are more employable: $f_1(\Theta) > f_2(\Theta)$

Then Proposition 4 implies $R_2^c(a_2, l_2; \Theta) > R_1^c(a_1, l_1; \Theta)$.

Table 1: International Comparison

	s	$f(\Theta)$	percentage of households owning equities	percentage of households owning risky financial assets	share of household portfolios in currency and deposits
United States	.019	.282	48.9	49.2	12.4
United Kingdom	.009	.056	31.5	32.4	26.0
Germany	.006	.035	18.9	25.1	33.9
France	.007	.033	–	–	29.1
Spain	.012	.020	–	–	38.1
Italy	.004	.013	18.9	22.1	27.9

Table 2: International Comparison

	s	$f(\Theta)$	$\frac{s+f(\Theta)}{r+s+f(\Theta)}$	Relative Risk Aversion R^c $\gamma = 2$ $\chi = 1.5$	$\gamma = 5$ $\chi = 0.5$	$\gamma = 10$ $\chi = 2.5$	$\gamma = 20$ $\chi = 10$
Theoretical labor market benchmarks:							
perfect rigidity	0	0	0	2	5	10	20
near-perfect flexibility	1	1	.997	0.86	0.46	2.01	6.68
International comparison, $r = .004$:							
United States	.019	.282	.977	0.87	0.46	2.04	6.77
United Kingdom	.009	.056	.903	0.91	0.50	2.17	7.13
Germany	.006	.035	.854	0.94	0.52	2.26	7.38
France	.007	.033	.851	0.94	0.53	2.27	7.40
Spain	.012	.020	.821	0.96	0.54	2.34	7.57
Italy	.004	.013	.708	1.03	0.62	2.61	8.28

Table 3: Cyclical Variation in Risk Aversion

					Relative Risk Aversion R^c			
					$\gamma = 2$	$\gamma = 5$	$\gamma = 10$	$\gamma = 20$
	s	$f(\Theta)$	r	$\frac{s+f(\Theta)}{r+s+f(\Theta)}$	$\chi = 1.5$	$\chi = 0.5$	$\chi = 2.5$	$\chi = 10$
U.S., expansion	.017	.35	.003	.995	0.86	0.46	2.01	6.70
U.S., recession	.022	.20	.011	.953	0.88	0.47	2.08	6.88
rigid lab mkt, expan	.0036	.016	.003	.868	0.93	0.52	2.24	7.31
rigid lab mkt, recess	.0046	.009	.011	.557	1.15	0.76	3.10	9.45

Other International Evidence: Campbell (1999)

	consumption growth std. dev. 1970–96	equity premium 1970-96
United States	0.9%	4.5%
United Kingdom	2.6%	6.2%
Germany	2.5%	4.8%
France	2.1%	4.5%
Netherlands	2.8%	9.0%
Switzerland	2.2%	10.1%
Sweden	1.9%	6.4%
Italy	1.7%	−1.5%

Conclusions

General conclusions:

- A flexible labor margin affects risk aversion
- Risk premia are closely related to risk aversion

Implications of labor market frictions:

- Risk aversion is higher in more frictional labor markets
- Risk aversion is higher in recessions
- Risk aversion is higher for households that are less employable

Quantitative findings:

- Frictions can play a contributing role to higher risk aversion in Europe
- Risk aversion formulas in Swanson (2012) still a good approximation