# Risk Aversion, Risk Premia, and the Labor Margin with Generalized Recursive Preferences

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# Coefficient of Relative Risk Aversion

Suppose a household has preferences:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t),$$

$$u(c_t, l_t) = \frac{c_t^{1-\gamma}}{1-\gamma} - \eta\, l_t$$

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Answer: 0

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What is the household's coefficient of relative risk aversion?

Answer: $\dfrac{1}{\frac{1}{\gamma} + \frac{1}{\chi}}$

# Empirical Relevance of the Labor Margin

Imbens, Rubin, and Sacerdote (2001):

- Individuals who win a lottery prize reduce labor supply by $.11 for every $1 won (note: spouse may also reduce labor supply)

Coile and Levine (2009):

- Older individuals are 7% less likely to retire in a given year after a 30% fall in stock market

Coronado and Perozek (2003):

- Individuals who held more stocks in late 1990s retired 7 months earlier

Large literature estimating wealth effects on labor supply (e.g., Pencavel 1986)

# Outline of Presentation

- Define risk aversion rigorously for expected utility preferences
- Show the labor margin can have big effects on risk aversion
- Generalize the results to Epstein-Zin preferences
- Discuss asset pricing examples

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See the paper for:

- More asset pricing details
- Numerical solutions far away from steady state
- Multiplier preferences

# A Household

Household preferences:

$$E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} u(c_\tau, l_\tau),$$

Flow budget constraint:

$$a_{\tau+1} = (1 + r_\tau) a_\tau + w_\tau l_\tau + d_\tau - c_\tau,$$

No-Ponzi condition:

$$\lim_{T\to\infty} \prod_{\tau=t}^{T} (1 + r_{\tau+1})^{-1} a_{T+1} \geq 0,$$

$\{w_\tau, r_\tau, d_\tau\}$ are exogenous processes, governed by $\theta_\tau$

# The Value Function

State variables of the household's problem are $(a_t; \theta_t)$.

Let:

$$c_t^* \equiv c^*(a_t; \theta_t),$$

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Value function, Bellman equation:

$$V(a_t; \theta_t) = u(c_t^*, l_t^*) + \beta E_t V(a_{t+1}^*; \theta_{t+1}),$$

where:

$$a_{t+1}^* \equiv (1 + r_t)a_t + w_t l_t^* + d_t - c_t^*.$$

# Technical Conditions

**Assumption 1.** *The function $u(c_t, l_t)$ is increasing in its first argument, decreasing in its second, twice-differentiable, and strictly concave.*

**Assumption 2.** *The value function $V : X \to \mathbb{R}$ for the household's optimization problem exists and satisfies the Bellman equation*

$$V(a_t; \theta_t) = \max_{(c_t, l_t) \in \Gamma(a_t; \theta_t)} u(c_t, l_t) + \beta E_t V(a_{t+1}; \theta_{t+1}).$$

**Assumption 3.** *For any $(a_t; \theta_t) \in X$, the household's optimal choice $(c_t^*, l_t^*)$ lies in the interior of $\Gamma(a_t; \theta_t)$.*

**Assumption 4.** *The value function $V(\cdot; \cdot)$ is twice-differentiable in its first argument. (It then follows that $c^*$, $l^*$ are differentiable.)*

# Assumptions about the Economic Environment

**Assumption 5.** *The household is* infinitesimal*.*

**Assumption 6.** *The household is* representative*.*

**Assumption 7.** *The model has a* nonstochastic steady state*,* $x_t = x_{t+k}$ *for* $k = 1, 2, \ldots,$ *and* $x \in \{c, l, a, w, r, d, \theta\}$*.*

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**Assumption 7′.** *The model has a* balanced growth path *that can be renormalized to a nonstochastic steady state after a suitable change of variables.*

# Arrow-Pratt in a Static One-Good Model (Review)

Compare:

$$E\,u(c + \sigma\varepsilon) \quad \text{vs.} \quad u(c - \mu)$$

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$$E\,u(c+\sigma\varepsilon) \;\approx\; u(c) + u'(c)\sigma E[\varepsilon] + \frac{1}{2}u''(c)\sigma^2 E[\varepsilon^2],$$

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Coefficient of absolute risk aversion is defined to be:

$$\lim_{\sigma\to 0} 2\mu(\sigma)/\sigma^2 \;=\; \frac{-u''(c)}{u'(c)}\,.$$

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Note $(*)$ is equivalent to gamble over asset returns:

$$a_{t+1} = (1 + r_t + \sigma\tilde{\varepsilon}_{t+1})a_t + w_t l_t + d_t - c_t.$$

or income:

$$a_{t+1} = (1 + r_t)a_t + w_t l_t + (d_t + \sigma\varepsilon_{t+1}) - c_t,$$

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Welfare loss from $\mu$:

$$V_1(a_t; \theta_t) \frac{\mu}{(1 + r_t)}$$

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Welfare loss from $\mu$:

$$\beta E_t V_1(a^*_{t+1}; \theta_{t+1})\,\mu.$$

Loss from $\sigma$:

$$\beta E_t V_{11}(a^*_{t+1}; \theta_{t+1})\frac{\sigma^2}{2}.$$

# Coefficient of Absolute Risk Aversion

**Definition 1.** *The household's coefficient of absolute risk aversion at* $(a_t; \theta_t)$ *is given by* $R^a(a_t; \theta_t) = \lim_{\sigma \to 0} 2\mu(\sigma)/\sigma^2$.

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**Proposition 1.** *The household's coefficient of absolute risk aversion at* $(a_t; \theta_t)$ *is well defined and given by*

$$R^a(a_t; \theta_t) \; = \; \frac{-E_t V_{11}(a^*_{t+1}; \theta_{t+1})}{E_t V_1(a^*_{t+1}; \theta_{t+1})} \, .$$

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**Proposition 1.** *The household's coefficient of absolute risk aversion at* $(a_t; \theta_t)$ *satisfies*

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*Evaluated at the nonstochastic steady state, this simplifies to:*

$$R^a(a; \theta) \;=\; \frac{-V_{11}(a; \theta)}{V_1(a; \theta)} \, .$$

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# Solve for $V_1$ and $V_{11}$

Benveniste-Scheinkman:

$$V_1(a_t;\theta_t) = (1+r_t)\,u_1(c_t^*, l_t^*). \qquad (*)$$

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$$V_1(a_t;\theta_t) = (1+r_t)\, u_1(c_t^*, l_t^*). \qquad (*)$$

Differentiate $(*)$ to get:

$$V_{11}(a_t;\theta_t) \;=\; (1+r_t)\left[u_{11}(c_t^*, l_t^*)\,\frac{\partial c_t^*}{\partial a_t} \;+\; u_{12}(c_t^*, l_t^*)\,\frac{\partial l_t^*}{\partial a_t}\right].$$

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Household Euler equation:

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Differentiate, substitute out for $\partial l_t^*/\partial a_t$, and use BC, TVC to get:

$$\frac{\partial c_t^*}{\partial a_t} \;=\; \frac{r}{1 + w\lambda}\,.$$

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**Proposition 2.** *The household's coefficient of absolute risk aversion in Proposition 1, evaluated at steady state, satisfies:*

$$R^a(a;\theta) \;=\; \frac{-V_{11}(a;\theta)}{V_1(a;\theta)} \;=\; \frac{-u_{11}+\lambda u_{12}}{u_1}\;\frac{r}{1+w\lambda}.$$

# Coefficient of Absolute Risk Aversion

**Corollary 3.**

$$R^a(a;\theta) = \frac{-u_{11} + \lambda u_{12}}{u_1} \frac{r}{1 + w\lambda} \leq \frac{-u_{11}}{u_1} r\,.$$

*If $r < 1$, then $R^a(a;\theta)$ is also less than $-u_{11}/u_1$.*

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$$u(c_t, l_t) = c_t^{\theta}(\bar{l} - l_t)^{1-\theta}.$$

# Relative Risk Aversion

Consider Arrow-Pratt gamble of general size $A_t$:

$$a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t + A_t \sigma \varepsilon_{t+1},$$

vs.

$$a_{t+1} = (1 + r_t)a_t + w_t l_t + d_t - c_t - A_t \mu.$$

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Risk aversion coefficient for this gamble:

$$\frac{-A_t E_t V_{11}(a^*_{t+1}; \theta_{t+1})}{E_t V_1(a^*_{t+1}; \theta_{t+1})}. \qquad (*)$$

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A natural benchmark for $A_t$ is household wealth at time $t$.

# Household Wealth

In DSGE framework, household wealth has more than one component:

- financial assets $a_t$
- present value of labor income, $w_t l_t$
- present value of net transfers, $d_t$
- present value of leisure, $w_t(\bar{l} - l_t)$?

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Leisure, in particular, can be hard to define, e.g.,

$$u(c_t, l_t) = \frac{c_t^{1-\gamma}}{1-\gamma} - \eta \frac{l_t^{1+\chi}}{1+\chi}$$

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and $\bar{l}$ is arbitrary.

Different definitions of household wealth lead to different definitions of relative risk aversion.

# Two Coefficients of Relative Risk Aversion

**Definition 2.** *The consumption-wealth coefficient of relative risk aversion, $R^c(a_t;\theta_t)$, is given by $(*)$ with* $A_t \equiv (1+r_t)^{-1} E_t \sum_{\tau=t}^{\infty} m_{t,\tau} c^*_\tau$.

In steady state:

$$R^c(a;\theta) = \frac{-A\,V_{11}(a;\theta)}{V_1(a;\theta)} = \frac{-u_{11}+\lambda u_{12}}{u_1}\,\frac{c}{1+w\lambda}.$$

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$$R^c(a;\theta) = \frac{-A\,V_{11}(a;\theta)}{V_1(a;\theta)} = \frac{-u_{11}+\lambda u_{12}}{u_1} \frac{c}{1+w\lambda}.$$

**Definition 3.** *The consumption-and-leisure-wealth coefficient of relative risk aversion*, $R^{cl}(a_t;\theta_t)$, *is given by* $(*)$ *with* $\widetilde{A}_t \equiv (1+r_t)^{-1} E_t \sum_{\tau=t}^{\infty} m_{t,\tau}\left(c_\tau^* + w_\tau(\bar{l} - l_\tau^*)\right)$.

In steady state:

$$R^{cl}(a;\theta) = \frac{-\widetilde{A}\,V_{11}(a;\theta)}{V_1(a;\theta)} = \frac{-u_{11}+\lambda u_{12}}{u_1} \frac{c+w(\bar{l}-l)}{1+w\lambda}.$$

# Household with Generalized Recursive Preferences

Household chooses state-contingent $\{(c^t, l^t)\}$ to maximize

$$V(a_t;\theta_t) \;=\; \max_{(c_t,l_t)\in\Gamma(a_t;\theta_t)} \; u(c_t,l_t) + \beta\left(E_t\, V(a_{t+1};\theta_{t+1})^{1-\alpha}\right)^{1/(1-\alpha)}$$

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Note: Generalized recursive preferences are often written as:

$$U(a_t;\theta_t) = \max_{(c_t,l_t)\in\Gamma(a_t;\theta_t)} \left[\widetilde{u}(c_t,l_t)^{\rho} + \beta \left(E_t\, U(a_{t+1};\theta_{t+1})^{\widetilde{\alpha}}\right)^{\rho/\widetilde{\alpha}}\right]^{1/\rho}$$

# Household with Generalized Recursive Preferences

Household chooses state-contingent $\{(c^t, l^t)\}$ to maximize

$$V(a_t;\theta_t) \; = \max_{(c_t,l_t)\in\Gamma(a_t;\theta_t)} \; u(c_t,l_t) + \beta \left( E_t \, V(a_{t+1};\theta_{t+1})^{1-\alpha} \right)^{1/(1-\alpha)}$$

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It's easy to map back and forth from $U$ to $V$; moreover,

- $V$ is more closely related to standard dynamic programming results, regularity conditions, and FOCs
- $V$ makes derivations, formulas in the paper simpler
- additively separable $u$ is easier to consider in $V$

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subject to flow budget constraint

$$a_{\tau+1} = (1+r_\tau)a_\tau + w_\tau l_\tau + d_\tau - c_\tau$$

and No-Ponzi condition.

$\{w_\tau, r_\tau, d_\tau\}$ are exogenous processes, governed by $\theta_\tau$.

State variables of the household's problem are $(a_t;\theta_t)$.

# Coefficient of Absolute Risk Aversion

**Proposition 1.** *The household's coefficient of absolute risk aversion at* $(a_t; \theta_t)$*, denoted* $R^a(a_t; \theta_t)$*, satisfies*

$$\frac{-E_t\left[V(a^*_{t+1};\theta_{t+1})^{-\alpha}V_{11}(a^*_{t+1};\theta_{t+1}) - \alpha\, V(a^*_{t+1};\theta_{t+1})^{-\alpha-1}V_1(a^*_{t+1};\theta_{t+1})^2\right]}{E_t\ V(a^*_{t+1};\theta_{t+1})^{-\alpha}V_1(a^*_{t+1};\theta_{t+1})}$$

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**Proposition 3.** *The household's coefficient of absolute risk aversion in Proposition 1, evaluated at steady state, satisfies:*

$$R^a(a;\theta) \;=\; \frac{-u_{11}+\lambda u_{12}}{u_1}\,\frac{r}{1+w\lambda} \;+\; \alpha\frac{r\,u_1}{u}\,.$$

# Asset Pricing

Expected excess return on asset $i$:

$$\begin{aligned} \psi_t^i &\equiv E_t r_{t+1}^i - r_{t+1}^f \\ &= -\text{Cov}_t(m_{t+1}, r_{t+1}^i) \end{aligned}$$

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**Proposition 7.** *To first order around the nonstochastic steady state,*

$$dm_{t+1} = -R^a(a;\theta)\, d\hat{A}_{t+1} + d\Phi_{t+1}$$

*To second order around the nonstochastic steady state,*

$$\psi_t^i = R^a(a;\theta)\, Cov_t(dr_{t+1}^i, d\hat{A}_{t+1}) - Cov_t(dr_{t+1}^i, d\Phi_{t+1})$$

# Numerical Example

Economy is a very simple, standard RBC model:

- Competitive firms
- Cobb-Douglas production, $y_t = Z_t k_t^{1-\zeta} l_t^{\zeta}$
- AR(1) technology, $\log Z_{t+1} = \rho_z \log Z_t + \varepsilon_t$
- Capital accumulation, $k_{t+1} = (1-\delta)k_t + y_t - c_t$
- Equity is a consumption claim
- Equity premium is expected excess return,

$$\psi_t = \frac{E_t(C_{t+1} + p_{t+1})}{p_t} - (1 + r_t^f)$$

# Numerical Example: Preferences

Period utility

$$u(c_t, l_t) = \frac{c_t^{1-\gamma}}{1-\gamma} - \eta \frac{l_t^{1+\chi}}{1+\chi}$$

Generalized recursive preferences

$$V(a_t;\theta_t) = \max_{(c_t,l_t)\in\Gamma(a_t;\theta_t)} u(c_t, l_t) + \beta \left( E_t\, V(a_{t+1};\theta_{t+1})^{1-\alpha} \right)^{1/(1-\alpha)}$$

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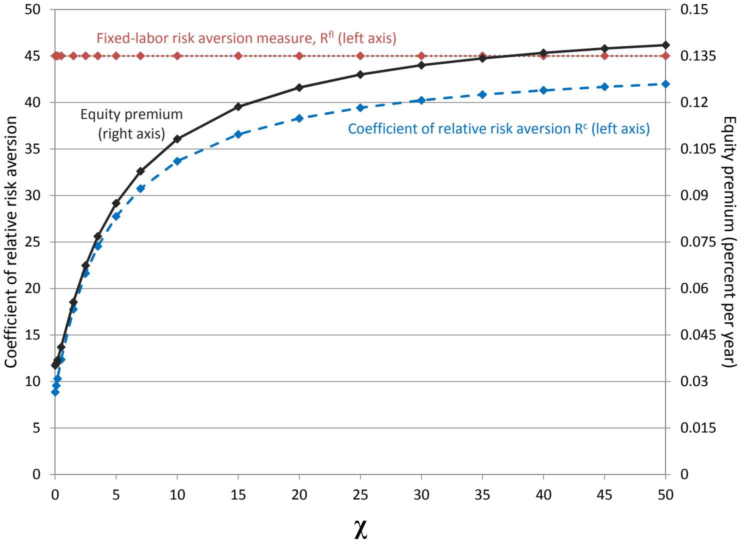
Note:

- IES $= 1/\gamma$
- If labor fixed, relative risk aversion is $R^{fl} = \gamma + \alpha(1-\gamma)$
- If labor flexible, relative risk aversion is $R^c$, depends on $\chi$, $\gamma$, $\alpha$

# Additively Separable Period Utility



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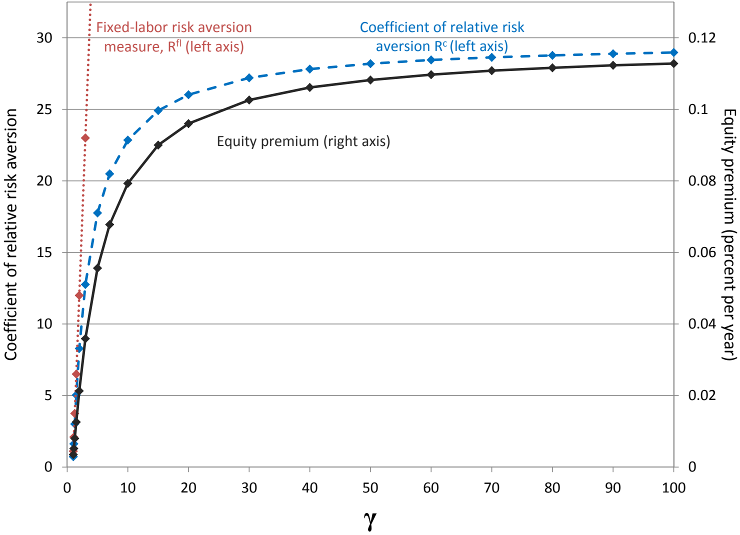


# Second Numerical Example

Same RBC model as before, with Cobb-Douglas period utility

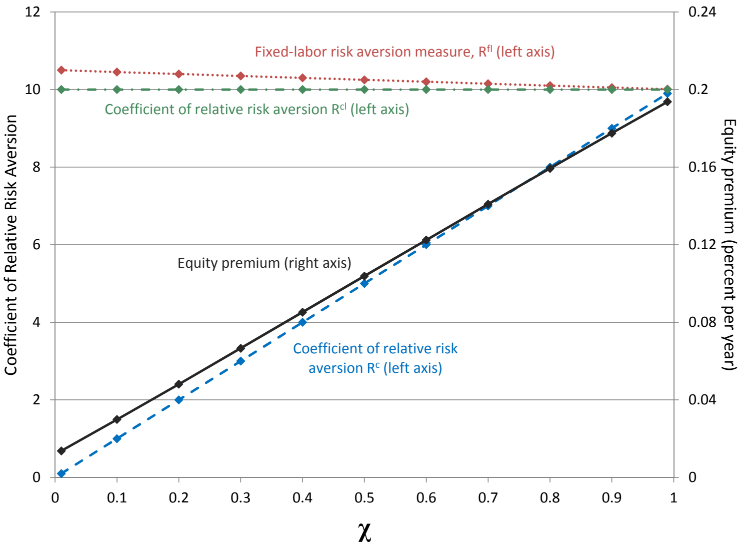
$$u(c_t, l_t) = \frac{\left(c_t^{\chi}(1-l_t)^{1-\chi}\right)^{1-\gamma}}{1-\gamma}$$

and random-walk technology, $\rho_z = 1$.

Note:

- IES $= 1/\gamma$
- If labor fixed, risk aversion is $R^{fl} = (1 - \chi(1-\gamma)) + \alpha(1-\gamma)$
- For composite good, risk aversion is $R^{cl} = \gamma + \alpha(1-\gamma)$
- Risk aversion $R^c$ recognizes labor is flexible, excludes value of leisure from household wealth, $R^c = \chi\gamma + \chi\alpha(1-\gamma)$

# Cobb-Douglas Period Utility



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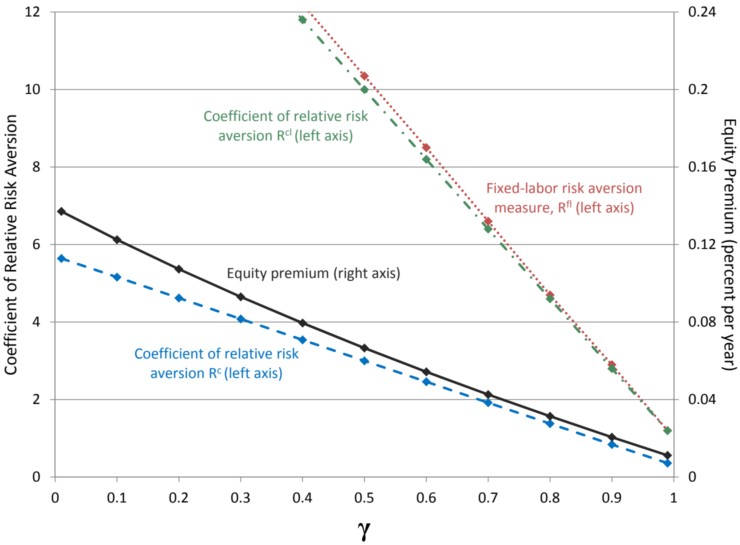

# Risk Neutrality

Hansen-Rogerson linear-labor preferences are common:

- Extensive labor margin: Hansen (1985), Rogerson (1988)
- Monetary search: Lagos-Wright (2005)
- Investment: Khan-Thomas (2008), Bachmann-Caballero-Engel (2010), Bachmann-Bayer (2009)

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The present paper suggests ways to model risk neutrality that do not require linear utility of consumption.

# Empirical Estimates of Risk Aversion

Barsky-Juster-Kimball-Shaprio (1997):

> "Suppose that you are the only income earner in the family, and you have a good job guaranteed to give you your current (family) income every year for life. You are given the opportunity to take a new and equally good job, with a 50–50 chance it will double your (family) income and a 50–50 chance that it will cut your (family) income by a third. Would you take the new job?"

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Empirical estimates of risk aversion using methods like these remain generally valid in the framework of the present paper, but should be phrased more carefully.

What is different is how these estimates are mapped into model parameters (i.e., risk aversion $\neq -cu_{11}/u_1$)

# Empirical Asset Pricing

Campbell (1996, 1999): $u(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma}, \quad m_{t+1} = \log \beta - \gamma \Delta c_{t+1}$

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| Country | $E_t(r_{e,t} - r_{f,t})$ | $\text{std}(r_{e,t} - r_{f,t})$ | $\text{std}(\Delta c)$ | $\gamma$ |
|---|---|---|---|---|
| USA | 5.82 | 17.0 | 0.91 | 37.3 |
| JPN | 6.83 | 21.6 | 2.35 | 13.4 |
| GER | 6.77 | 20.4 | 2.50 | 13.3 |
| FRA | 7.12 | 22.8 | 2.13 | 14.6 |
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If $u(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma} - \eta\frac{L_t^{1+\chi}}{1+\chi}$, then $\gamma \neq$ risk aversion.

# Conclusions

1. A flexible labor margin affects risk aversion
2. Risk premia are related to risk aversion
3. Fixed-labor measure of risk aversion performs poorly
4. Composite-good measure of risk aversion also seems to perform poorly
5. For multiplier preferences, risk aversion is very sensitive to scaling by $(1-\beta)$
6. Simple, closed-form expressions for risk aversion with:
    - flexible labor margin
    - generalized recursive preferences
    - external or internal habits
    - validity away from steady state
    - correspondence to risk premia in the model
7. New paper: frictional labor markets