The Bond Premium in a DSGE Model with Long-Run Real and Nominal Risks

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Part of a Broader Project

- Rudebusch, Sack, Swanson (2006): term premium in standard NK DSGE models is far too small, stable relative to the data.

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- this paper: Epstein-Zin preferences in a NK DSGE model

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Relative to equity premium, the term premium:

- only requires modeling short-term interest rate, not dividends or leverage
- is used by central banks to measure expectations of monetary policy, inflation
- applies to a larger volume of securities
- provides an additional perspective on the model
- tests nominal rigidities

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DSGE model:

- many empirical questions about risk premia require a structural DSGE model to provide reliable answers
- DSGE models widely used in macroeconomics; total failure to explain risk premia may signal flaws in the model

Epstein-Zin Preferences

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We'll use standard NK utility kernel:

$$u(c_t, l_t) \equiv \frac{c_t^{1-\gamma}}{1-\gamma} - \chi_0 \frac{l_t^{1+\chi}}{1+\chi}$$

Epstein-Zin Preferences

Household optimality conditions with EZ preferences:

$$\begin{aligned}
\mu_t \, u_1\big|_{(c_t,l_t)} &= P_t\lambda_t \\
-\mu_t \, u_2\big|_{(c_t,l_t)} &= w_t\lambda_t \\
\lambda_t &= \beta E_t \lambda_{t+1}(1+r_{t+1}) \\
\mu_t &= \mu_{t-1}\left(E_{t-1} V_t^{1-\alpha}\right)^{\alpha/(1-\alpha)} V_t^{-\alpha}, \quad \mu_0 = 1
\end{aligned}$$

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Recall: $V_t = u(c_t, l_t) + \beta \big(E_t V_{t+1}^{1-\alpha} \big)^{1/(1-\alpha)}$

The DSGE Model

- Continuum of households with Epstein-Zin preferences
  - consume output, supply labor
- Continuum of Dixit-Stiglitz differentiated firms
  - set prices in Calvo contracts with avg. duration 4 quarters
  - identical Cobb-Douglas production functions
  - face aggregate technology: $\log A_t = \rho_A \log A_{t-1} + \varepsilon_t^A$
- Government
  - purchases G_t, financed by lump-sum taxes
  - $\log G_t = \rho_G \log G_{t-1} + (1 - \rho_g) \log \bar{G} + \varepsilon_t^G$
- Monetary Authority
  - sets short-term nominal interest rate using a Taylor-type rule
  - monetary policy shock

The Term Premium in the Model

Asset pricing:

$$p_t = d_t + E_t[m_{t+1} p_{t+1}]$$

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Term premium:

$$\psi_t^{(n)} \equiv i_t^{(n)} - \hat{i}_t^{(n)}$$

Solving the Model

State variables of the model:

$$A_{t-1},\, G_{t-1},\, i_{t-1},\, \bar{\pi}_{t-1},\, \Delta_{t-1},\, \varepsilon_t^A,\, \varepsilon_t^G,\, \varepsilon_t^i$$

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We solve the model by perturbation methods

- We compute a *third*-order approximation of the solution around nonstochastic steady state
- Perturbation AIM algorithm in Swanson, Anderson, Levin (2006) quickly computes *n*th order approximations

Result: Model Fits Basic Macro, Finance Moments

Table 2: Empirical and Model-Based Unconditional Moments

Variable	U.S. Data 1961–2007	EU Preferences	EZ Preferences	"best fit" EZ Preferences
sd[C]	1.19	1.40	1.46	2.12
sd[L]	1.71	2.48	2.50	1.89
sd[w^r]	0.82	2.02	2.02	2.02
sd[π]	2.52	2.22	2.30	2.96
sd[i]	2.71	1.86	1.93	2.65
sd[$i^{(40)}$]	2.41	0.52	0.57	1.17
mean[$\psi^{(40)}$]	1.06	.010	.438	1.06
sd[$\psi^{(40)}$]	0.54	.000	.053	.162
mean[$i^{(40)} - i$]	1.43	$-$.038	.390	0.95
sd[$i^{(40)} - i$]	1.33	1.41	1.43	1.59
mean[$x^{(40)}$]	1.76	.010	.431	1.04
sd[$x^{(40)}$]	23.43	6.52	6.87	10.77
memo: IES		.5	.5	.5
quasi-CRRA		2	75	90

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Coefficient of Relative Risk Aversion

- Epstein-Zin preferences:

$$m_{t,t+1} \equiv \frac{\beta u_1\big|_{(c_{t+1},l_{t+1})}}{u_1\big|_{(c_t,l_t)}} \left(\frac{V_{t+1}}{\left(E_t V_{t+1}^{1-\alpha}\right)^{1/(1-\alpha)}} \right)^{-\alpha} \frac{P_t}{P_{t+1}}$$

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- Guvenen (2006), Moskowitz-Vissing-Jorgensen (2009): heterogeneous agents

Long-Run Inflation Risk

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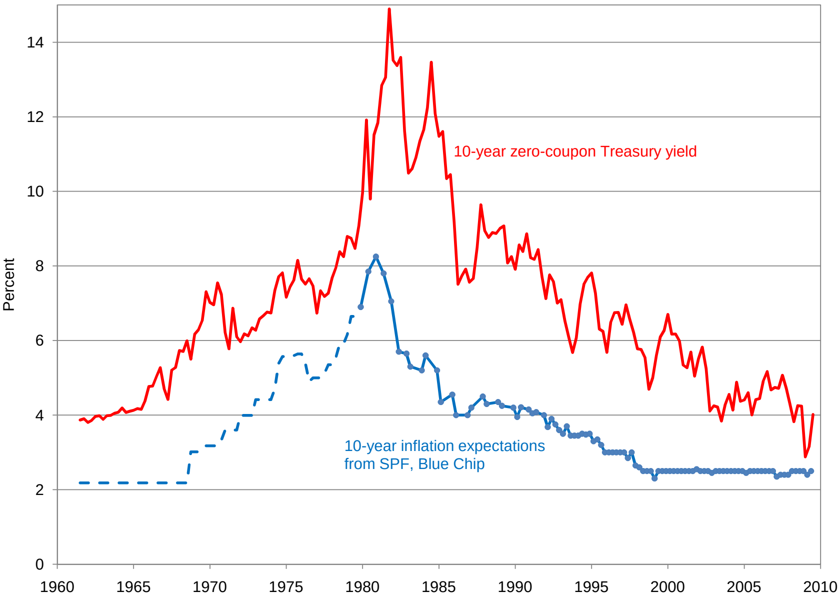
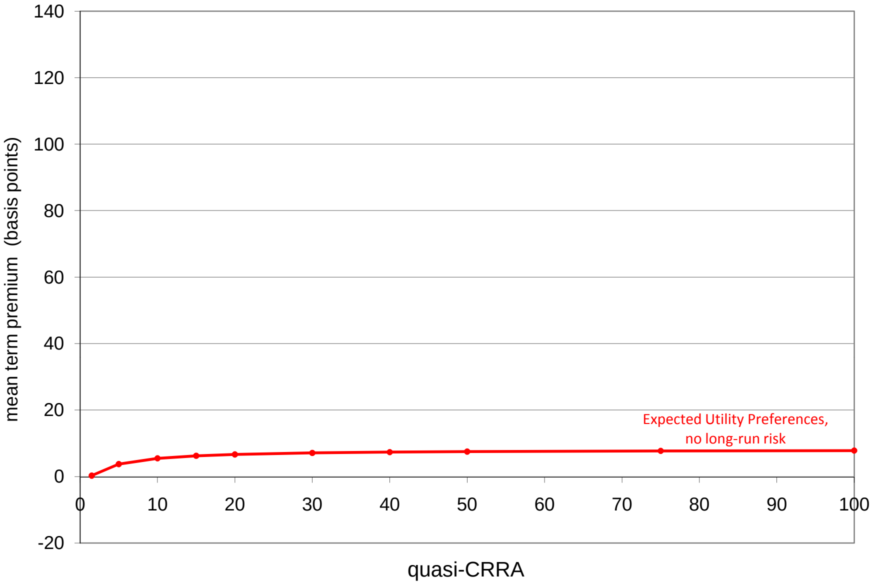
Long-run inflation risk makes long-term bonds more risky:

- same idea as Bansal-Yaron (2004), but with nominal risk rather than real risk
- long-term inflation expectations more observable than long-term consumption growth
- other evidence (Kozicki-Tinsley, 2003, Gürkaynak, Sack, Swanson, 2005) that long-term inflation expectations in the U.S. vary

Long-Run Inflation Risk



Long-Run Inflation Risk and the Term Premium



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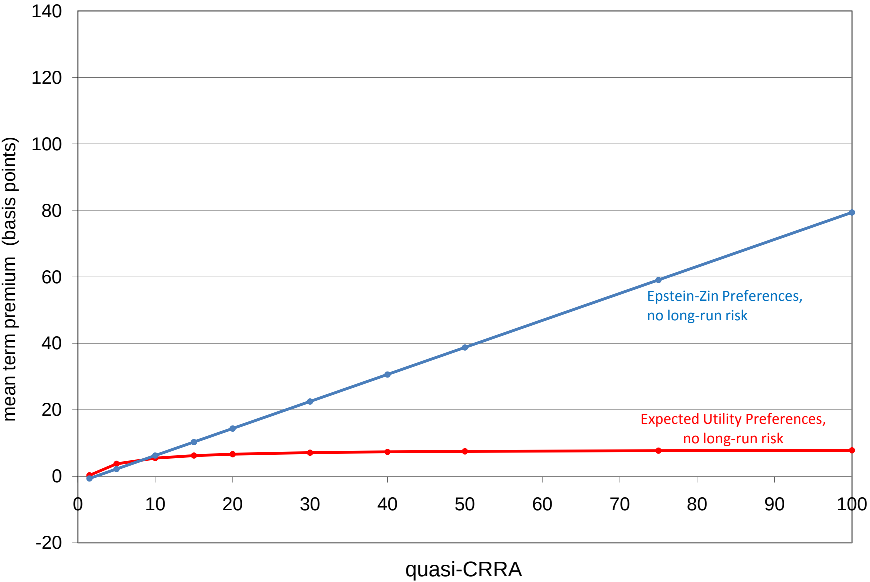

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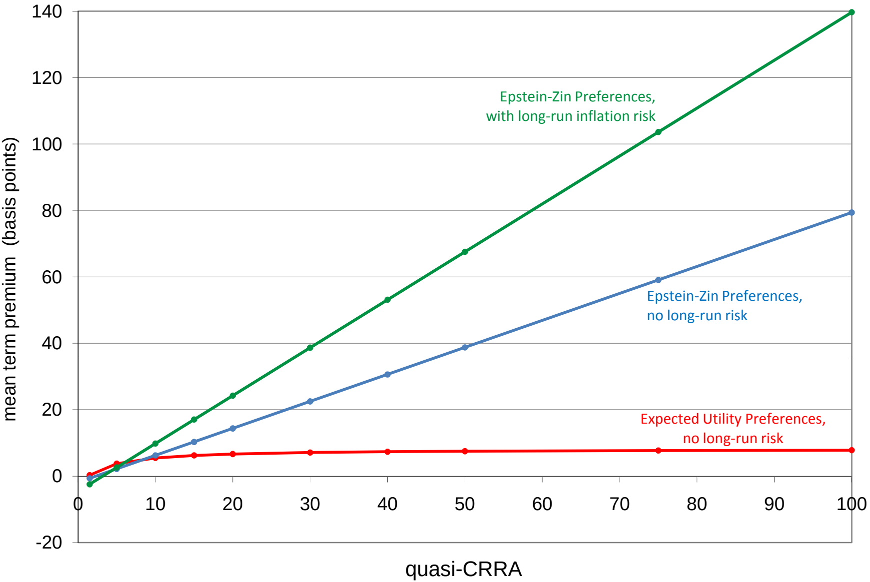

Result: Nominal Yield Curve is Upward-Sloping

Backus-Gregory-Zin (1989), Den Haan (1995)

- if interest rates are low in recessions
- then bond prices rise in recessions
- $\implies$ the term premium should be negative
- the yield curve slopes downward

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Note: Backus et. al intuition still applies to real yield curve

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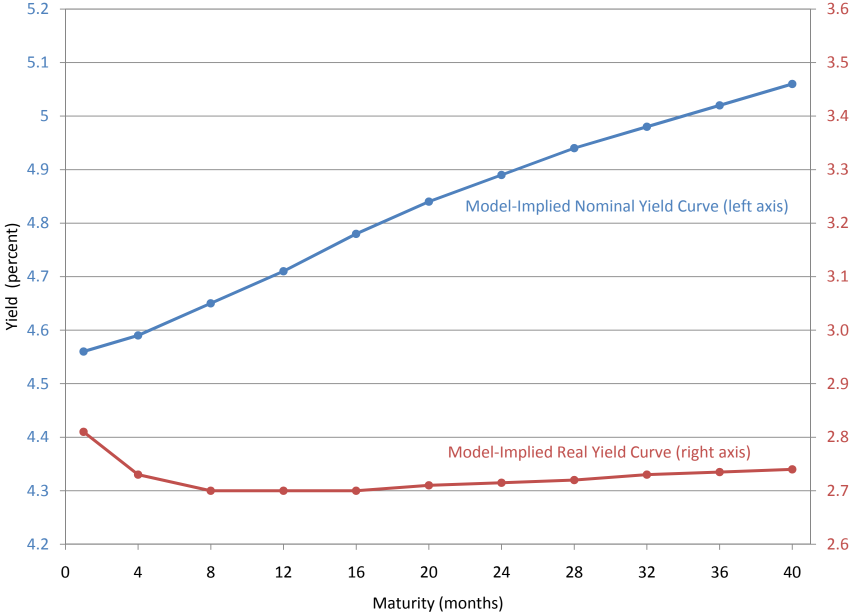

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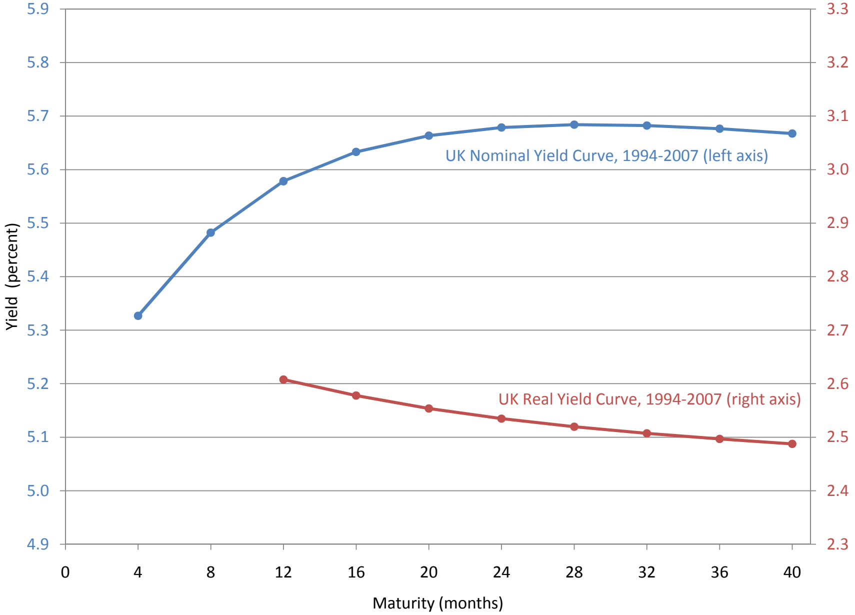

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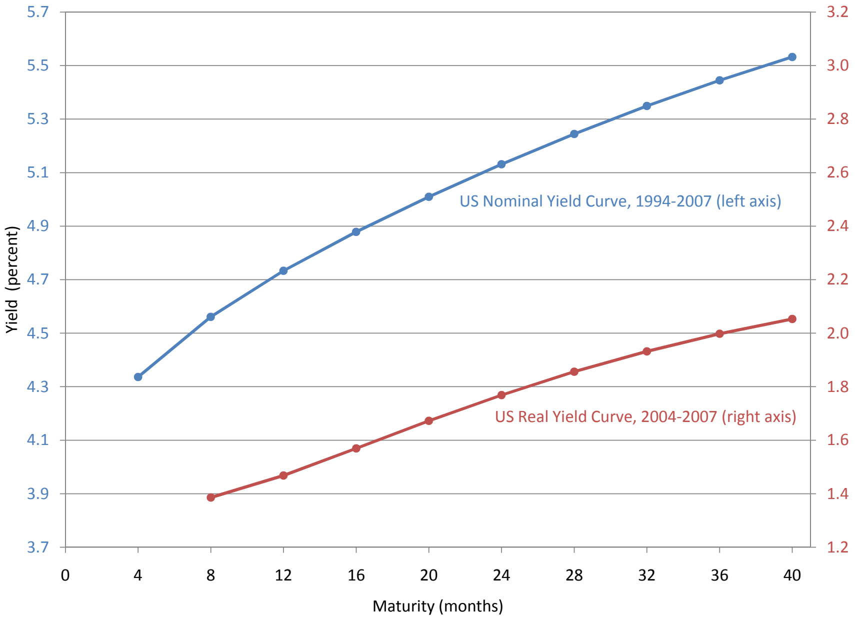
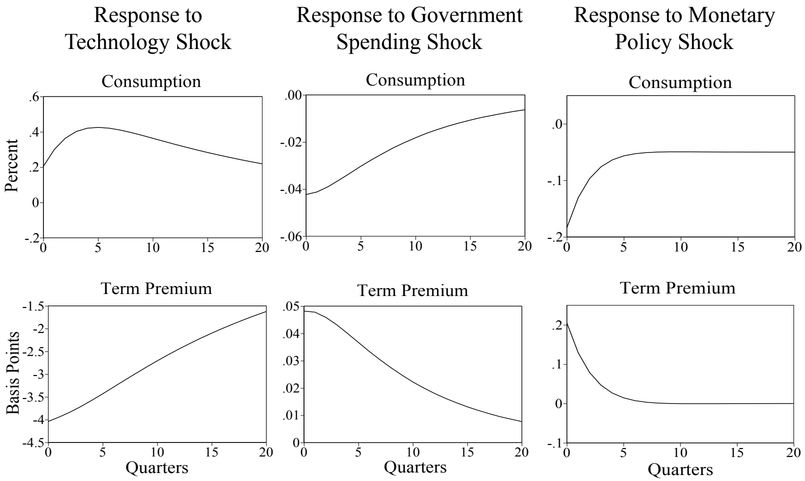


Result: Model Term Premium is Countercylical



Result: Model Generates Endogenous Heterosked.

$$p_t^{(2)} - \hat{p}_t^{(2)} = E_t m_{t+1} p_{t+1}^{(1)} - E_t m_{t+1} E_t p_{t+1}^{(1)} = Cov_t(m_{t+1}, p_{t+1}^{(1)})$$

time-varying term premium $\iff$ conditional heteroskedasticity

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Second-order solution:

$$\begin{aligned} x_t &= \mu_x + \sum \alpha_x dx_{t-1} + \sum \alpha_\varepsilon \varepsilon_t \\ &+ \sum \alpha_{xx} dx_{t-1} dx_{t-1} + \sum \alpha_{x\epsilon} dx_{t-1} \varepsilon_t + \sum \alpha_{\varepsilon\varepsilon} \varepsilon_t \varepsilon_t + \ldots \end{aligned}$$

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Model	term premium mean (bp)	term premium std dev (bp)
baseline model	86.5	11.0
log-linear log-normal	86.5	0.0

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3. Epstein-Zin preferences can solve bond premium puzzle in endowment economy, are much more promising in NK DSGE framework:
   agents are risk-averse and cannot offset long-run real or nominal risks
4. Long-run risks reduce the required quasi-CRRA, increase volatility of risk premia, help fit financial moments