Examining the Bond Premium Puzzle with a DSGE Model

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Outline

The Bond Premium Puzzle

The equity premium puzzle: excess returns on stocks are much larger (and more variable) than can be explained by standard preferences in a DSGE model (Mehra and Prescott, 1985).

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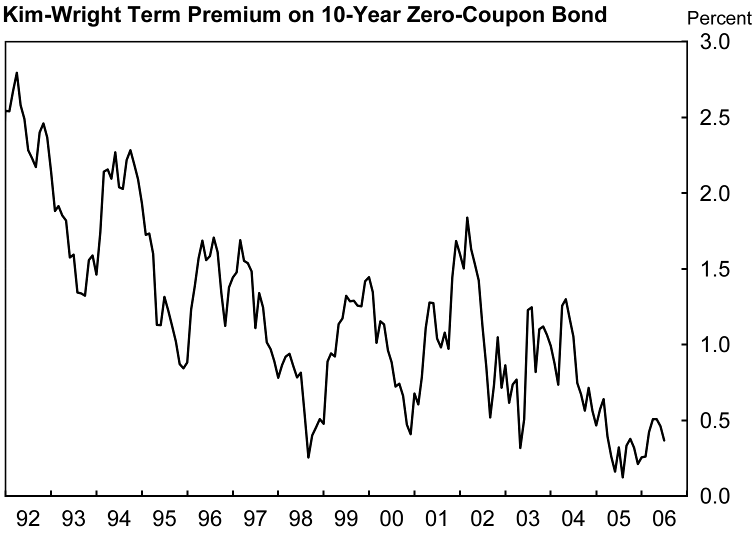
Note:

- Since Backus, Gregory, and Zin (1989), DSGE models with nominal rigidities have advanced considerably

Kim-Wright Term Premium



Why Study the Bond Premium Puzzle?

The bond premium puzzle is important:

- DSGE models increasingly used for policy analysis; total failure to explain term premium may signal flaws in the model
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The equity premium puzzle has received more attention in the literature, but the bond premium puzzle:

- provides an additional perspective on the model
- tests nominal rigidities in the model
- only requires modeling short-term interest rate process, not dividends or leverage
- applies to a larger volume of U.S. securities

Recent Studies of the Bond Premium Puzzle

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  - can resolve bond premium puzzle using Campbell-Cochrane preferences in endowment economy

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  - the term premium is very small in a standard, simple calibrated New Keynesian model

Moreover, in the present paper, we show:

- in the Christiano, Eichenbaum, Evans (2006) model, term premium is 1 bp

The Term Premium in a Benchmark DSGE Model

2. The Term Premium in a Benchmark New Keynesian Model
   - Define Benchmark New Keynesian Model
   - Review Asset Pricing
   - Solve the Model

Benchmark New Keynesian Model (Very Standard)

Representative household with preferences:

$$\max E_t \sum_{t=0}^{\infty} \beta^t \left(\frac{(c_t - h_t)^{1-\gamma}}{1-\gamma} - \chi_0 \frac{l_t^{1+\chi}}{1+\chi} \right)$$

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$$m_{t+1} = \frac{\beta(C_{t+1} - bC_t)^{-\gamma}}{(C_t - bC_{t-1})^{-\gamma}} \frac{P_t}{P_{t+1}}$$

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Parameters: $\beta = .99$, $b = .66$, $\gamma = 2$, $\chi = 1.5$

Benchmark New Keynesian Model (Very Standard)

Continuum of differentiated firms:

- face Dixit-Stiglitz demand with elasticity $\frac{1+\theta}{\theta}$, markup θ
- set prices in Calvo contracts with avg. duration 4 quarters
- identical production functions $y_t = A_t \bar{k}^{1-\alpha} l_t^{\alpha}$
- have firm-specific capital stocks
- face aggregate technology $\log A_t = \rho_A \log A_{t-1} + \varepsilon_t^A$

Parameters $\theta = .2$, $\rho_A = .9$, $\sigma_A^2 = .01^2$

Perfectly competitive goods aggregation sector

Benchmark New Keynesian Model (Very Standard)

Government:

- imposes lump-sum taxes G_t on households
- destroys the resources it collects
- $\log G_t = \rho_G \log G_{t-1} + (1 - \rho_g) \log \bar{G} + \varepsilon_t^G$

Parameters $\bar{G} = .17\bar{Y}$, $\rho_G = .9$, $\sigma_G^2 = .004^2$

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Monetary Authority:

$$i_t = \rho_i i_{t-1} + (1-\rho_i)\left[1/\beta + \pi_t + g_y(y_t - \bar{y}) + g_\pi(\bar{\pi}_t - \pi^*)\right] + \varepsilon_t^i$$

Parameters $\rho_i = .73$, $g_y = .53$, $g_\pi = .93$, $\pi^* = 0$, $\sigma_i^2 = .004^2$

Asset Pricing

Asset pricing:

$$p_t = d_t + E_t[m_{t+1} p_{t+1}]$$

Zero-coupon bond pricing:

$$p_t^{(n)} = E_t[m_{t+1} p_{t+1}^{(n-1)}]$$

$$i_t^{(n)} = -\frac{1}{n} \log p_t^{(n)}$$

Notation: let $i_t \equiv i_t^{(1)}$

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Term premium:

$$\psi_t^{(n)} \equiv \log\left(\frac{\delta_c \widetilde{p}_t^{(n)}}{\widetilde{p}_t^{(n)} - 1}\right) - \log\left(\frac{\delta_c \widehat{p}_t^{(n)}}{\widehat{p}_t^{(n)} - 1}\right)$$

Solving the Model

The benchmark model above has a relatively large numer of state variables: C_{t-1}, A_{t-1}, G_{t-1}, i_{t-1}, Δ_{t-1}, $\bar{\pi}_{t-1}$, ε_t^A, ε_t^G, ε_t^i

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We solve the model by approximation around the nonstochastic steady state (perturbation methods)

- In a first-order approximation, term premium is zero
- In a second-order approximation, term premium is a constant (sum of variances)
- So we compute a *third*-order approximation of the solution around nonstochastic steady state
- Perturbation AIM algorithm in Swanson, Anderson, Levin (2006) quickly computes *n*th order approximations

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- shocks in macro models have standard deviations $\approx .01$
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To make these higher-order terms important,

- need “high curvature” modifications from finance literature
- or shocks with standard deviations $\gg .01$

Robustness of Results

Table 1: Alternative Parameterizations of Baseline Model

Parameter	Baseline case value	Low case value	Low case mean[ψ_t]	High case value	High case mean[ψ_t]
γ	2	.5	-1.5	6	4.5
χ	1.5	0	.6	5	2.9
b	.66	0	1.0	.9	2.6
ρ_A	.9	.7	.4	.95	3.9
σ_A^2	$.01^2$	$.005^2$	.6	$.02^2$	4.7
ρ_i	.73	0	3.8	.9	.7
g_π	.53	.05	-3.5	1	3.3
g_y	.93	0	3.5	2	-1.0
π^*	0	0	–	.02	2.1

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Models with Giant Shocks

Hördahl, Tristani, Vestin (2006) match level of term premium using:

- NK model very similar to our benchmark model
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Ravenna and Seppälä (2007) match level of term premium using:

- NK model similar to above
- preferences: $\dfrac{(c_t - bC_{t-1})^{1-\gamma}}{1-\gamma} - \xi_t \chi_0 \dfrac{l_t^{1+\chi}}{1+\chi}$
- giant preference shocks: $\rho_\xi = .95$, $\sigma_\xi = .08$
- in our benchmark model, imply consol term premium of 19.7bp

Models with Giant Shocks

Table 3: Unconditional Moments

		Parameterizations of DSGE Model		
Variable	U.S. Data	Baseline	HTV	RS
sd[C]	1.19	1.36	12.5	5.14
sd[Y]	1.50	0.86	7.90	3.24
sd[L]	1.71	2.81	9.73	5.14
sd[w^r]	0.82	2.27	12.6	10.7
sd[π]	2.52	2.35	15.3	7.67
sd[i]	2.71	2.06	15.1	7.02
sd[$i^{(10)}$]	2.37	0.55	10.2	2.70
mean[$\psi^{(10)}$]	1.06	.014	.686	.197
sd[$\psi^{(10)}$]	0.54	.001	1.51	.081
mean[$i^{(10)} - i$]	1.43	$-$.050	.651	.171
sd[$i^{(10)} - i$]	2.30	1.55	5.37	4.55
mean[$x^{(10)}$]	1.76	$-$.038	.684	.193
$\beta_{CS}^{(10)}$	$-$3.49	0.96	0.98	1.00

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Slow-Moving Habits and Labor Market Frictions

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   - Campbell-Cochrane Habits
   - Campbell-Cochrane Habits with Labor Market Frictions

Campbell-Cochrane Habits

Preferences: $$\frac{(c_t - H_t)^{1-\gamma}}{1-\gamma} - \chi_0 \frac{l_t^{1+\chi}}{1+\chi}$$

Habits defined implicitly by $S_t \equiv \dfrac{C_t - H_t}{C_t}$, where:

$$\log S_t = \phi \log S_{t-1} + (1-\phi)\log \bar{S} + \frac{1}{\bar{S}}\left(\sqrt{1 - 2(\log S_{t-1} - \log \bar{S})} - 1\right)(\Delta \log C_t - E_{t-1}\Delta \log C_t)$$

Campbell-Cochrane calibrate $\phi = .87$, $\bar{S} = .0588$

Campbell-Cochrane Habits: Results

Recall: Wachter (2005) resolves bond premium puzzle using:

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- endowment economy
- random walk consumption
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Intuition: in a DSGE model, households can self-insure by varying labor supply

Campbell-Cochrane Habits and Labor Market Frictions

Possible solution:

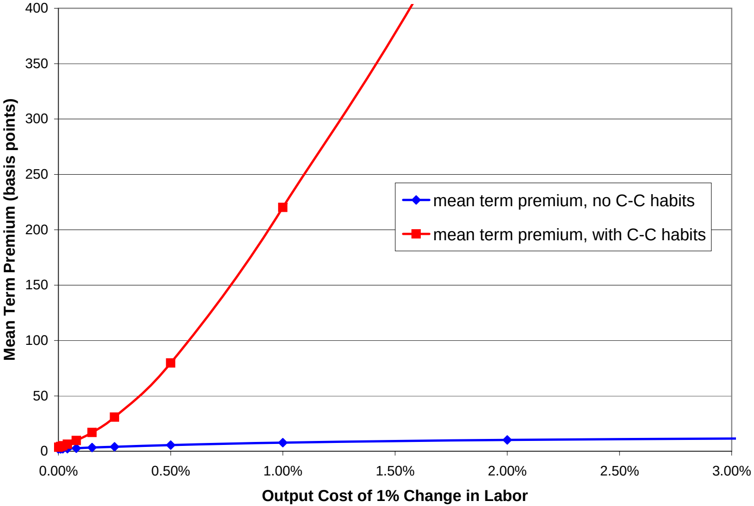
- add labor market frictions to prevent households from self-insuring

Explore three classes of labor market frictions:

- households pay an adjustment cost: $\kappa(\log l_t - \log l_{t-1})^2$
- staggered nominal wage contracting
- real wage rigidities (Nash bargaining)

Campbell-Cochrane Habits with Adjustment Costs

Figure 1: Mean Term Premium



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sd[Y]	0.86	0.71	0.59
sd[L]	2.81	2.88	3.60
sd[w^r]	2.27	2.14	220.9
sd[π]	2.35	2.25	19.7
sd[i]	2.06	2.05	7.66
sd[$i^{(10)}$]	0.55	0.57	1.19
mean[$\psi^{(10)}$]	.014	.027	.640
sd[$\psi^{(10)}$]	.001	.001	.095
mean[$i^{(10)} - i$]	$-$.050	$-$.046	.593
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Intuition: complete markets provide households with insurance, more than offsets the costs of the wage friction

Real Wage Rigidities

Following Blanchard and Galí (2005), model real wage bargaining rigidity as:

$$\log w_t^r = (1-\mu)\big(\log w_t^{r*} + \omega\big) + \mu \log w_{t-1}^r$$

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Intuition: wage friction increases volatility of MRS, but decreases volatility of inflation, interest rates

Additional Robustness Checks

- estimation, “best fit” parameters
- larger models (CEE, LOWW)
- models with investment
- internal habits
- markup shocks
- time-varying π_t^*

None of these have helped to fit the term premium

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4. There appears to be no easy way to fix this in the standard, habit-based NK DSGE framework
5. Ongoing work: Epstein-Zin preferences

Epstein-Zin-Weil Preferences

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Three key ingredients:

1. Nominal rigidities
   - makes bond pricing interesting
2. Epstein-Zin-Weil preferences
   - makes households risk averse
3. Long-run inflation risk
   - introduces a risk households cannot control
   - makes bonds risky

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Note:

- need to impose $u \geq 0$
- or $u \leq 0$ and $V_t \equiv u(c_t, l_t) - \beta \left(E_t (-V_{t+1})^{\alpha} \right)^{1/\alpha}$

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$$V_t \equiv u(c_t, l_t) + \beta \left(E_t V_{t+1}^{\alpha}\right)^{1/\alpha}$$

Note:

- need to impose $u \geq 0$
- or $u \leq 0$ and $V_t \equiv u(c_t, l_t) - \beta \left(E_t(-V_{t+1})^{\alpha}\right)^{1/\alpha}$

We'll use standard NK utility kernel:

$$u(c_t, l_t) \equiv \frac{{c_t}^{1-\gamma}}{1-\gamma} - \chi_0 \frac{l_t^{1+\chi}}{1+\chi}, \tag{1}$$

Epstein-Zin-Weil Preferences

Household optimality conditions with EZW preferences:

$$\begin{aligned}
\mu_t \, u_1|_{(c_t, l_t)} &= P_t \lambda_t \\
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\lambda_t &= \beta E_t \lambda_{t+1}(1 + r_{t+1}) \\
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Stochastic discount factor:

$$m_{t,t+1} \equiv \frac{\beta u_1|_{(c_{t+1},l_{t+1})}}{u_1|_{(c_t,l_t)}} \left(\frac{V_{t+1}}{(E_t V_{t+1}^{\alpha})^{1/\alpha}} \right)^{1-\alpha} \frac{P_t}{P_{t+1}}$$

Epstein-Zin-Weil Preferences

Long-run inflation risk:

$$\pi_t^* = \rho_{\pi^*}\pi_{t-1}^* + (1 - \rho_{\pi^*})\theta_{\pi^*}(\overline{\pi}_t - \pi_t^*) + \varepsilon_t^{\pi^*}$$

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