SEPARATUM

DOCUMENTA OPHTHALMOLOGICA

ADVANCES IN OPHTHALMOLOGY

VOL. XVIII

Proceedings of the Symposium of Physiology of Flicker and of the 2nd Symposium of the International Society for Clinical Electroretinography (ISCERG) on Flicker Electroretinography, September 1963

LINEAR THEORY AND THE PSYCHOPHYSICS OF FLICKER

by

George Sperling
(Murray Hill, N.J., U.S.A.)

With 4 figures

Dr. W. JUNK, PUBLISHERS
THE HAGUE

1964

LINEAR THEORY AND THE PSYCHOPHYSICS OF FLICKER

by

GEORGE SPERLING
(Murray Hill, N.J., U.S.A.)

With 4 figures

INTRODUCTION

The study of the psychophysics of flicker is currently experiencing a renaissance. New analytic and experimental developments are having a powerful impact and we are fortunate in having obtained the leaders in the field for this session. You should be cautioned, however, that four of the speakers have been trained as physicists and the remaining pair are mathematically inclined physiologists; their contributions will be not only enlightening but also quite technical. In order to facilitate their presentations, I shall provide some technical background by reviewing briefly the linear analytic approach to flicker. Let me begin from the beginning by posing three questions: What is the psychophysics of flicker? Wherein lie its special difficulties? and What is the aim of the linear approach?

Psychophysics of Flicker

We are indebted to FECHNER (1860) for the word psychophysics. Today, as then, the *physics* half of the word refers to a physical description of a stimulus. In the case of flicker, the stimulus may be described in terms of a pattern of illumination on the retina, some portion of the illumination being turned on and off or having its intensity modulated with time.

The *psycho-* half of psychophysics originally referred to the assumed effect of the stimulus upon the psyche or consciousness of the observer. Psychophysicists no longer argue whether or not they can study consciousness but usually are satisfied to study verbal and other easily measured behavioral responses. Historically, the overwhelming majority of flicker studies have dealt with but a single kind of response: the observer indicated whether or not a light appeared to be flickering. Also investigated, although to a lesser extent, were other responses to indicate the apparent rate of flicker, the amount of flicker, the apparent brightness of a flickering source, and a few others.

It should be noted, parenthetically, that because of substantial variability from trial to trial and time to time, responses require statistical treatment. A highly precise modern methodology of psychophysics has developed to cope with such statistical situations; it relies particularly on forced choice judgments and such. But because so many flicker phenomena are unsolved, expediency has dictated that in studies of flicker such refinements of psychophysical methodology should be neglected – I think justifiably so.

Therefore the psychophysics of flicker, as we have it today, is simply the study of a few kinds of response to flickering stimuli. Its task is to predict the response, given the stimulus. The task of the physiologist, ultimately, is to account for the psychophysical relations in terms of physiological units.

The Problem

The difficulty in the psychophysics of flicker arises not from the small number of permissible responses but from the staggeringly large number of different possible stimuli. Some of the possible dimensions along which stimuli may vary are spatial dimensions[1] (such as the size or shape of a flickering stimulus, retinal location, the presence of other stimuli), intensity, wavelength composition, prior adaptation of the eye, the shape of the wave form, and the frequency of repetition for periodic stimuli. Nonperiodic stimuli have hardly been studied.

Some of these dimensions are more troublesome than others – for example waveform – because it is not clear how to extrapolate from one waveform to another; i.e., from results obtained with square waves, to those with triangular waves, pulses, sine waves, etc. The spatial variables are similarly difficult because of the difficulty in extrapolating systematically from one spatial pattern of stimulation to another.

The most reasonable and significant attempt at a solution is to find a set of basic waveforms with the property that every other waveform can be expressed as a combination of the basic waveforms. The aim is to collect data with the basic waveforms and then use that data to predict everything else. Essentially, this attempt is the linear approach to flicker.

---

[1] The number of visual spatial dimensions is infinity x infinity (not two). A complete specification of a monocular visual stimulus must include its intensity as a function of time at each of an infinite number of horizontal and vertical coordinates.

It is well known to physical scientists that every physically realizable waveform can be expressed mathematically in terms of a set of basic waveforms. Different basic sets are appropriate in different cases; in the case of periodic signals it is the set of sine waves.

In our sister science, psychoacoustics, the analysis of the acoustic signal into its sine wave components dates back to the last century. Shortly after FOURIER developed the mathematical techniques, OHM proposed in 1843 that the ear hears the separate sine wave components of a complex acoustic stimulus. This now classical formulation is known as Ohm's psychoacoustic law.[2]

In flicker, IVES (1922a, b) and COBB (1934a, b) urged the use of sinusoids, but they themselves did not carry out sufficient experiments to make the sinusoids fully useful. It was not until about 10 years ago when DE LANGE of Holland (1952, 1958a, b) used a sinusoidally modulated stimulus and studied intensively the conditions under which it appeared to flicker, that sinusoidal analysis came to fruition. Ideally, other stimuli can be analyzed into their component sine waves, the effect of each of these components can be determined from graphs or tables, and the separate results then combined into a prediction of flicker or no flicker for the complex stimulus. DE LANGE did not treat the question of combination extensively because in many, if not most, of the usual cases one particular sine wave predominates so completely that the effects of the others, whatever they might reasonably be expected to be, can be neglected.

Some of the other flicker stimulus variables may also be amenable to linear analysis. KELLY (1960) proposed the use of Bessel functions for spatial stimuli having radial symmetry. For the effect of wavelength, Drs. WALRAVEN and LEEBEEK in this symposium will be using PITT's (1944) three basic trichromatic filter functions just as we all have used the red, green, and blue C.I.E. curves.

PROPERTIES OF LINEAR SYSTEMS

An analysis into sine waves or other set of basic functions is a linear analysis. The question of the appropriateness or inappropriateness of such an analysis *is* the question of linearity or nonlinearity of a system. The question of linearity is fundamental in the psychophysics of flicker – or any other system – because while linear systems can be analyzed

---

[2] See, for example, BORING (1942).

routinely, there is no guarantee that a nonlinear system can be analyzed at all, by any method. Therefore it is appropriate to examine in detail what a linear system is, the relations of sine waves to linear systems, and of linear systems to flicker vision. The exposition is in the form of a series of questions and answers.

1. What is a linear system?

The terminology *input-system-output* is exactly analogous to, but more general than, the terminology *stimulus-observer-response.* A system is a set of relations between quantities called inputs and outputs; it may be a written set of rules, a physical machine, or an organism. For example, we shall be talking about the system: light → ERG (flickering light is the input, the ERG is the output) and the system: flickering light → verbal response ("it flickers" or "it is fused").

A linear system is one in which the input-output relations are described by linear equations. The linear property is equivalent to a simple additive principle called superposition.

> Let an input a cause a response A and an input b cause a response B. The system is linear if the compound input $a + b$ causes a response of exactly $A + B$ for every choice of a and b.

The superposition property makes even complicated systems readily analyzable. It means that a stimulus may be considered as divided into components, each of the components traced through the system separately, and the separate outputs then recombined to yield the total output.

2. What is a sinusoidally modulated stimulus?

An unmodulated and a sinusoidally modulated stimulus are illustrated in Fig. 1. The abscissa represents time.

The ordinate represents light intensity. The left part of Fig. 1 illustrates a light of intensity L_o; the right part of the figure illustrates a light of intensity L_o which is being sinusoidally modulated by an amount $m \times L_o$ at a frequency of f cycles per second. Such a picture would be produced by displaying on an oscilloscope the output of a photocell which is monitoring the stimulus. The mathematical representation of the waveforms is indicated in the figure. Note that m is a pure number, indicating the per cent modulation. In Fig. 1, $m = 0.25$ (25%).

Except for daylight, modulated light sources are the rule rather than the exception. Artificial sources such as fluorescent and incandescent lamps which operate on alternating current produce approximately sine

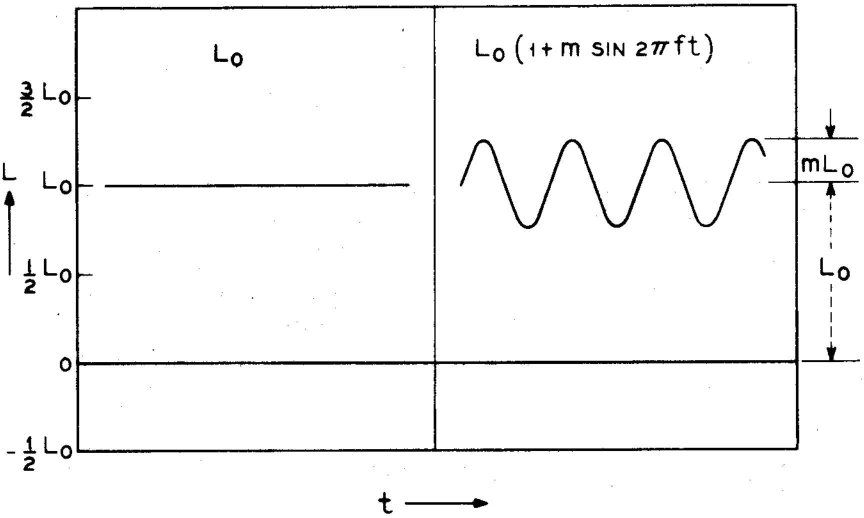

Fig. 1.

Unmodulated (left) and sinusoidally modulated (right) stimuli. Ordinate: relative luminance units (proportion of L_0). Abscissa: time.

wave modulated light. We usually do not notice the flicker in these sources because the frequency of modulation, f, is too high.

3. Why are sine waves uniquely suited to the study of linear systems? If the input to a linear system is a sine wave, then the output also must be a sine wave of the same frequency. For example, if the relation between modulated light stimuli and the ERG were linear, it must follow that a sinusoidally modulated light produces a pure sinusoidally modulated ERG. Depending upon the light modulation frequency, the ERG might be shifted in phase or, it might be of a larger or smaller amplitude. But it must be sinusoidal.

In this respect, sinusoids are the unvarying elements in a linear system. They pass through it unchanged except for a delay (phase shift) or attenuation. No compound waveform has this property. Inputs modulated as square waves or triangular waves usually produce outputs that are neither square nor triangular but some other shape, the particular output shape depending upon the system and the modulation frequency. Only a sine wave input inevitably reproduces its shape in the output of a linear system.

4. What is the advantage of using sine waves in the study of *unknown* systems? For example, suppose the eye is stimulated by flickering light

and the ERG is measured. Why is it desirable to use a sine wave modulated stimulus?

The answer is in two parts.

(1) If the system is linear, the response to a sine wave will be a sine wave. The knowledge of the amplitude and the phase shift of the response at each frequency constitutes complete knowledge of the system. The information for sine waves is perfectly adequate to predict the response to any other stimulus whatsoever. The complex stimulus is conceptually analyzed into its component sine waves, each of these is traced through the system, and the component outputs then recombined into the total output.

(2) If the system is not linear, it is probable that the nonlinearity will reveal itself as a failure of the output to replicate the sine wave input. The response to a sine wave might not be sinusoidal or it might be of a different frequency than the input. Once such a nonlinearity is revealed, of course, the system must be studied in other ways.

Thus, the advantage of using sine wave modulated stimuli is that if a system is linear, the analysis is completed; if it is not linear, the sine wave stimulus generally reveals this fact better than any other stimulus.

5. Suppose a system is not linear. Then what?

There are several possibilities. A transformation of variables may linearize it. For example, measuring voltage instead of power. However, in biological systems, the problem usually is more basic.

It may be possible to find a restricted range of the variables within which the system behaves linearly. One uses that range as a starting point and works out from it. DE LANGE's procedure was of this kind. He studied the visual system at a constant adaptation level and with only a small modulation of the stimulus around that level. Under these conditions he reasoned that the visual system would respond linearly. At a different adaptation level the visual system responds differently, but when the modulation m is restricted to small values, the response is still linear.

The technique of using signals so small that departures from linearity are insignificant is an extremely useful one. It works for nearly all systems and is a technique used in many of the papers to follow. When even this technique fails, it brings us to the third possibility: essential nonlinearity – the system never appears to respond linearly. Practically, one may include under this designation systems which behave linearly, but only for such infinitesimal ranges of the input that linear analysis has no utility.

Essentially-nonlinear systems can be very difficult to analyze because there are no systematic ways to unravel them. But it is very important to establish that a system is essentially nonlinear (cf. SPERLING, 1963). The experimenter then knows he must rely on his ingenuity more than on his training.

APPLICATION TO THE PSYCHOPHYSICS OF FLICKER

For the system: flickering light → ERG, the application of the linear principles is obvious. But the psychophysical system typically consists of a flickering light input and a response output of either "yes, it is flickering" or "no, it is fused." Such a system can be represented by two components: a linear and a nonlinear component. The linear component is called a filter; when its input is a sine wave, its output is a sine wave. The nonlinear component is called a detector; its input is the output of the filter and its output is either "yes" (if its input amplitude exceeds a fixed threshold) or "no" (if its input fails to exceed a fixed threshold). There relations are illustrated in Fig. 2.

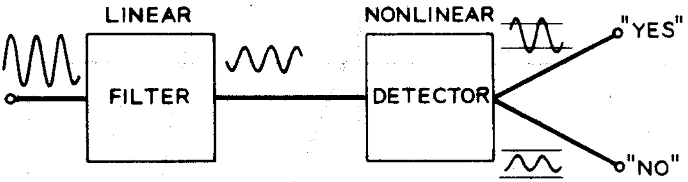

Fig. 2.
Schematic model for flicker-fusion judgments.

In this kind of model, linear and nonlinear elements work together, but because frequency independence was implicitly assumed for the detector, all the frequency dependent effects must occur in the linear filter. One can, of course, devise more complicated models, with more complicated components. Some inadequacies and necessary refinements of the present model will be discussed in subsequent papers. However, all models for visual processes will have in common the interplay between linear and nonlinear elements, which is one of the most interesting and perplexing theoretical problems in vision.

We are now ready to look at some actual psychophysical data and to see how it can be interpreted in terms of the model. IVES was the first to study the per cent of modulation, m, observers required in order to detect flicker as a function of the frequency, f. (See Fig. 1.) Figure 3 shows a graph of m versus f taken from more comprehensive data published by DE LANGE (1958a). In linear systems, similar graphs with logarithmic coordinates have special significance in their interpretation. Therefore Fig. 3 was graphed with logarithmic coordinates.

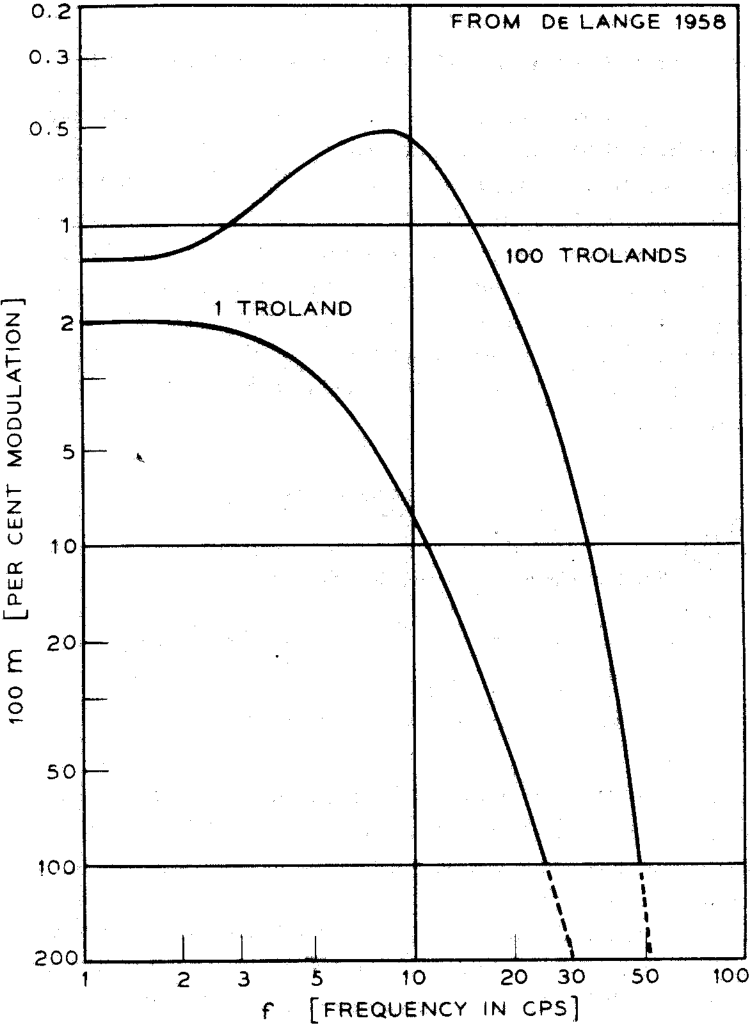

Fig. 3.

De Lange characteristics for human vision, 2° central field, white light. Threshold for detection of sinusoidal modulation m as a function of frequency f. From DE LANGE, 1958a, p. 782, Fig. 5.

The method of determining the per cent modulation needed to detect flicker is very intriguing, experimentally. One can measure the sensitivity to flicker at a particular adaptation level, L_0, and at a particular frequency, f. In the old methods, to pass from flicker to fusion, it was necessary to change either adaptation or frequency, rather than simply reducing the flicker *per se*.

The experimental data can be interpreted in terms of the two stage model as representing the attenuation characteristic of the filter component. For example, in viewing 10 cps sinusoidal flicker, an observer needs 0.5% modulation of a 100 troland light to detect flicker (Fig. 3). At 30 cps, 5% modulation is required. Both of these stimuli are just at threshold. Therefore, in terms of the model, the amplitude of the output of the linear filter must be the same. Because ten times more sine wave modulation was put into the filter at 30 cps than at 10 cps to achieve the same output, it follows that the filter component attenuates 30 cps ten times more than 10 cps.

Thus DE LANGE's data also can be interpreted as giving the attenuation characteristic of the linear filter, attenuation equaling the reciprocal of the input per cent modulation, m. The great explanatory power of this concept will be apparent subsequently. In honor of DE LANGE's experimental and theoretical contributions, I second the proposals of VAN DER TWEEL and LEVINSON that we call attenuation characteristics obtained with modulated light (as in Fig. 3) de Lange characteristics.

Equivalent RC-Stages

One further concept will be important for understanding the subsequent papers. The linear filter component can be specified in terms of an equivalent number of unit components. (A sequence of linear components is itself a linear component.) Each such unit is called an "RC-stage" because of its equivalence to an electrical circuit having resistance and capacitance elements. Figure 4 illustrates the output of systems having 1, 3, 5 and 10 identical successive, independent RC-stages when the input is a brief pulse (impulse). (When the input is a sine wave, the output is a sine wave, of course.)

The far right of Fig. 4 illustrates the ratio of a sine wave output to a sine wave input as a function of frequency. The filter attenuation characteristic for sine waves is the de Lange characteristic of the system (in the case of a threshold detector). It can be seen that increasing the number of identical RC-stages causes systematic changes in the attenuation char-

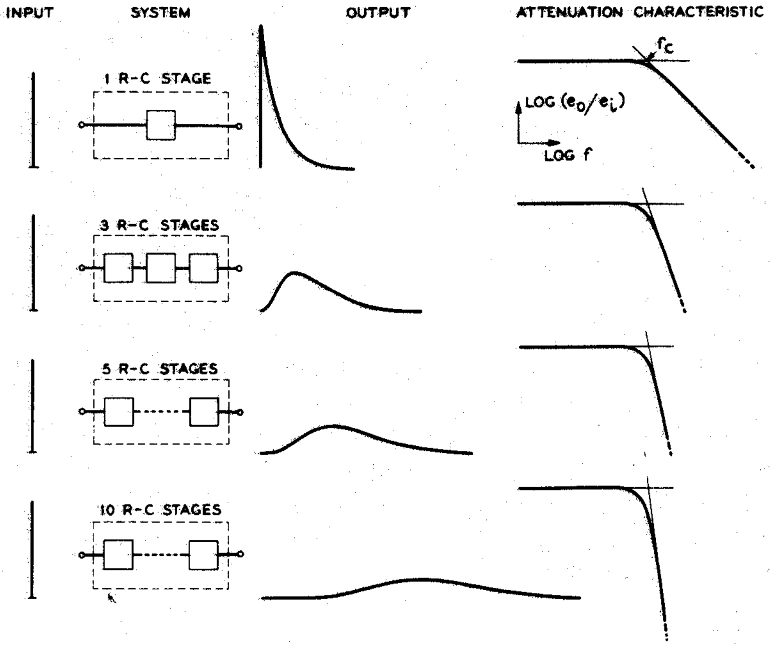

Fig. 4.

Comparison of systems consisting of 1, 3, 5, and 10 identical, successive RC-stages. The input is the same for each system, an impulse illustrated schematically at far left. The output is illustrated in center; coordinates are the same for each system. Attenuation is defined as e_o/e_i, where $e_i(= mL_o)$ is the amplitude of a sine wave input and e_o is the amplitude of the corresponding sine wave output. Graphs of log attenuation versus log frequency for each system are illustrated on far right.

acteristics of the filter. The final asymptote (diagonal line) in Fig. 4 becomes steeper but the intersection of the two lines (corresponding to the "cut-off frequency", f_c) does not change.

Figure 4 illustrates that by examining the de Lange characteristic of a system, and noting the steepness of the final asymptote, one can estimate the equivalent number of RC-stages. One of the results in the subsequent papers will be that several methods yield the same estimate of the number of RC-stages in flicker vision, namely about 10.

The greater the number of RC-stages, the greater the "spreading out" of an impulse. One of the interesting things to note in Fig. 4 is that while the time delay to the peak of the response is proportional to the number minus one of RC-stages, the actual delay of the onset of response is zero in all cases. The response begins building up immediately upon stimulation – although extremely slowly if there are many RC-stages.

Phase Delay

A de Lange characteristic does not give any phase information (unless so-called "minimum phase" conditions can be assumed to apply). An RC-stage analogy implies a particular set of phase as well as amplitude characteristics. Therefore, unless phase information also is obtained, one cannot be sure that a sequence of RC-stages represents a truly analogous system, even if the attenuation characteristics were to correspond exactly (which they usually do not). Phase information becomes important with complex inputs because the peak-to-peak amplitude of the output will depend on the relative phase delay (cf. LEVINSON, 1960).

An RC-analogy cannot be valid for an entire de Lange characteristic because the de Lange characteristic tends to diminish at low frequencies (particularly at high intensities) instead of remaining constant as required by the RC-analogy. However, the concept of equivalent RC-stages is familiar and remains useful as a means of describing data, particularly at high frequencies.

CONCLUDING REMARKS

Because the nonlinear detector of the model is frequency independent, the bulk of the "work" is done by the linear part of the model, hence the model often is loosely called a "linear" model. Even so, it should be noted that DE LANGE proposed linearity only for modulations small enough to be at the threshold of fusion, and so long as adaptation level was not varying. Although the linear property gives a model great simplicity, it behooves us to note that the linear approximations of today will yield to more precise, more complex, nonlinear approximations of tomorrow.

I ask your pardon for the didactic nature of my remarks. But be reminded that IVES' pioneering work lay dormant for 30 years until DE LANGE, because the scientists who should have profited from his ideas did not understand them. It is ironical that after publication of more than 1000 papers listed in LANDIS's (1953) bibliography on flicker, it still was not possible to answer such a basic question as "Under what conditions can flicker vision be approximated by a linear model?" Under such conditions – and they soon will be discussed in detail by the subsequent speakers – the results of a relatively small number of experiments with sine waves predict the results obtained with hundreds of configurations of rectangular and other of the waveforms that have been studied.

The subtle interplay of linear and nonlinear processes will be an important theme in the subsequent papers. I hope you will agree it was worth the attempt to provide a background.

Summary

The linear approach to the psychophysics of flicker attempts to predict the response to a flickering stimulus by analyzing the stimulus into elemental components. This leads naturally to the study of sinusoidal stimulation and of equivalent linear systems (filters). Filters are introduced and their unique relations to sinusoids are explained. Detection and adaptation are not linear processes, thus theories for flicker vision must include linear and nonlinear processes.

Résumé

Le traitement linéaire des problèmes psycho-physiques posés par le papillotement représentent une tentative visant à prévoir la réponse à un stimulus répététif en le décomposant en fractions élémentaires. Ceci mène naturellement à l'étude des stimulations conformes à une loi sinusoïdale et à celle des systèmes linéaires équivalents (filtres). On présente certains filtres, dont on explique les rapports spéciaux avec les sinusoïdes. La détection et l'adaption ne sont pas des processus linéaires. En conséquence, les théories de la vision en présence d'un papillotement doivent faire intervenir des processus linéaires et non-linéaires.

Zusammenfassung

Die lineare Behandlung der psychophysikalischen Probleme der Flimmerwahrnehmung versucht das Ansprechen auf einen Flimmerreiz dadurch vorauszubestimmen, dass dieser Reiz analysiert. bezw. in seine Elementarkomponenten zerlegt wird. Das führt natürlich zur Untersuchung der sinusförmigen Reizung und äquivalenter linearer Systeme (Filter). Es werden Filter eingeführt und deren eigenartige Beziehungen zu Sinuskurven erklärt. Wahrnehmung und Adaptation sind keine linearen Vorgänge; deshalb muss die Theorie des Flimmer-Sehens sowohl lineare wie auch nicht-lineare Vorgänge umfassen.

REFERENCES

BORING, E. G. (1942) Sensation and Perception in the History of Experimental Psychology. Appleton-Century Crofts: New York.

COBB, P. W. (1934a) Some Comments on the Ives Theory of Flicker. *J. opt. Soc. Amer.*, 24, *91–98*.

—— (1934b) The Dependence of Flicker on the Dark-Light Ratio of the Stimulus Cycle. *J. opt. Soc. Amer.*, 24, *107–113.*

FECHNER, G. T. (1860) Elemente der Psychophysik. Leipzig: Breitkopf & Härtel.

IVES, H. E. (1922a) Critical Frequency Relations in Scotopic Vision. *J. opt. Soc. Amer.*, 6, *253–268.*

—— (1922b) A Theory of Intermittent Vision. *J. opt. Soc. Amer.*, 6, *343–361.*

KELLY, D. H. (1960) Stimulus Patterns for Visual Research. *J. opt. Soc. Amer.*, 50, *1115–1116.*

LANDIS, C. (1953) An Annotated Bibliography of Flicker Fusion Phenomena. Covering the Period 1740/1952. Armed Forces NRC Vision Committee Secretariat, U. Michigan: Ann Arbor.

DE LANGE DZN, H. (1952) Relationship Between Critical Flicker-Frequency and a Set of Low Frequency Characteristics of the Eye. *J. opt. Soc. Amer.*, 44, *380–389.*

——, (1958a) Research into the Dynamic Nature of the Human Fovea-Cortex Systems with Intermittent and Modulated Light. I. Attenuation Characteristics with White and Colored Light. *J. opt. Soc. Amer.*, 48, *777–784.*

—— (1958b) Research into the Dynamic Nature of the Human Fovea-Cortex Systems with Intermittent and Modulated Light. II. Phase Shift in Brightness and Delay in Color Perception. *J. opt. Soc. Amer.*, 48, *784–789.*

LEVINSON, J. (1960) Fusion of Complex Flicker II. *Science*, 131, *1438–1440.*

OHM, G. S. (1843) Ueber die Definition des Tones, nebst daran geknüpfter Theorie der Sirene und ähnlicher tonbildender Vorrichtungen. *Ann. Phys. Chem.*, 135, *497–565.*

PITT, F. H. G. (1944) The Nature of Normal Trichromatic and Dichromatic Vision. *Proc. Roy. Soc.*, 132G, *101–117.*

SPERLING, G. (1963) Essential Nonlinearity of Visual Masking by Flickering Light. *J. opt. Soc. Amer.*, 53, *520–521.*

Bell Telephone Laboratories, Inc.