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Critical duration, supersummation, and the narrow domain of strength-duration experiments. *Overview.* Critical duration (as derived from strength-duration experiments) and supersummation are defined. An example demonstrates that supersummation can occur in a simple linear system when the criterion response property is that an output remains above threshold for a specified duration. In general, supersummation occurs when there is intensity compression or, equivalently, a time-limited process. Historically, critical duration has been studied as a means for deriving systematic understanding. When the general shape of the system impulse response is already known (e.g., that it is monophasic), the critical duration can provide a useful description of the parametric changes that occur with factors such as adaptation level, spectral composition of the test flash and of the background, and so forth. Unfortunately, strength-duration experiments are intrinsically inadequate for defining system transfer functions; for this, more complex paradigms – such as those involving multiple pulses or sine waves – are required.

This didactic note is in response to W & K's provocative but not entirely satisfactory article on critical duration. Since W & K do not provide a correct definition of *critical duration* we begin at the very beginning.

Stimuli. The classical concept of critical duration is defined for very restricted classes of stimuli – stimuli that vary only in strength and duration – that is, stimuli whose intensity i(t) is varied as a function of time so that

$$i(t) = \begin{cases} I & 0 \le t \le T \\ 0 & \text{elsewhere} \end{cases}$$

That is, the stimulus is *turned on* only for a time interval T and is *turned off* at all other times. We shall call such stimuli "pulses of intensity I and duration T," or "(I, T) pulses." For convenience we designate the time t_0 at which a stimulus is turned on as zero ($t_0 = 0$).

Responses. The response that the experimenter chooses to measure to an (I, T) pulse may be almost anything. Some numerical value or some property of this response is chosen by the experimenter as the *criterion response property* (usually abbreviated simply to *response, R*). Suppose the stimulus is an (I, T) pulse of light. Some examples of criterion response properties R are:

1. In a physiological preparation where the experimenter records a voltage $V(t)$ as a function of time: the criterion response property R could be selected to be a fixed peak voltage, such as max $\{V(t)\} = 2$ volts; or it could be min $\{V(t)\} = -0.5$ mV, or max − min = 3 mV, or the smallest time value t_{min} such that $V(t_{min}) \ge 2.3$ mV is 0.1 s (latency), or the longest time value t_{max} such that $V(t_{max}) \ge 2.3$V is 5.6 s, or $t_{max} - t_{min} = 2$ s (duration), or $V(t)$ has exactly two relative maxima; etc.
2. In a physiological preparation in which action potentials are measured: R could be that ten nerve impulses are recorded within one second of stimulus onset, or that exactly 10,000 nerve impulses are recorded in the total response to the stimulus.
3. In a psychophysical experiment with blocks of trials consisting of 100 (I, T) pulses and 100 blanks, randomly mixed: R could be exactly 65 correct target detections by an observer with exactly 5 false detections of blanks.
4. In the detection experiment such as (3) above: R could be a mean reaction time (on the correct detection trials) of exactly 300 ms.
5. In a search experiment for a target numeral embedded in a field of letters; R could be correct target identification on 50 percent of trials.
6. In a counting experiment: R could be a correct response on 65 percent of trials with stimuli containing exactly j elements (when the probability that stimuli contain exactly j elements is given by a particular probability distribution:
$$\rho(k), \sum_{k=0}^{N} \rho(k) = 1$$
7. In a reading experiment with flashed text: R could be answering 70 percent of the subsequent comprehension questions correctly.

In principle the I and T values of the stimulus pulses are varied so as to yield a large number of different (I, T) pairs, each of which produces the criterion response property. In practice this is feasible only for physiological preparations, and even there it is inefficient, because most stimuli do not produce precisely the criterion response property. However, the (I, T) pairs that would have produced the criterion response property usually can be inferred from pairs that produce too little or too much response. This inefficiency in data collection and utilization is a weakness of the critical-duration methodology. With I and T regarded as independent variables, and the observed amount of the criterion response as the dependent variable, one could do a more efficient and powerful data analysis. For example, the strength-duration data fall within the framework of conjoint measurement (Krantz & Tversky, 1970) and of monotonic analysis of variance (Kruskal, 1965). These are powerful descriptive and analytic tools that have not yet been adequately exploited by workers in this area. Moreover, they deal with the essential stochastic nature of the data, which is ignored in the traditional approach.

Critical Duration. Let us assume we have or can obtain (I, T) data pairs as needed, each of which produces the criterion response property. Let the energy $E(T)$ of any (I, T) pulse be defined as $E(T) = I(T) \cdot T$. (We need to write $I(T)$ as a function of T to keep track of which I goes with which T.) Suppose the limit of $E(T)$ as T goes to zero exists: let

$$E_0 = \lim_{T \to 0} E(T)$$

Consider the I value of (I, T) pairs as T goes to infinity:

$$\lim_{T \to \infty} I(T) = I_\infty$$

The critical duration T_c is the ratio of these two limits:

$$T_c = E_0/I_\infty$$

What purpose does this definition serve? In certain simple cases it is approximately true that $I \cdot T = E_0$ for $T \leq T_c$, and that $I = I_\infty$ for $T \geq T_c$. Thus, T_c (together with E_0 or I_∞) is a succinct characterization of a family of data. This relation is conveniently expressible in linear form on log-log coordinates (see Fig. 1). In human visual psychophysics this approximation as a determinant of visual threshold is sometimes taken as axiomatically true (for example, Graham & Kemp 1938; Ikeda & Boynton 1962) and sometimes as axiomatically false [as in the study of stimulus quantum fluctuations (Barlow 1957; 1958) and of probability summation resulting from "multiple looks" (Blackwell 1963)].

In spite of its long history in experimental psychology, there is seldom interest in critical duration itself; critical duration is nearly always studied for what it can reveal about the properties of the system. Critical duration is a means, not an end.

The simplest model with which we can reasonably hope to describe a physiological preparation or a psychological situation is a *linear system.* Linear systems are completely characterized by their impulse response (i.e., by their response to an impulse – an extremely brief pulse) or by their response to sine-wave stimuli (Schwartz & Friedland 1965; Sperling 1964). Thus, linear systems are studied most effectively with sine-wave stimuli and impulses. What can a strength-duration experiment tell us about the transfer function of a linear system? In fact, it has often been taken for granted that strength-duration experiments with (I, T) pulses are inherently inadequate to determine the transfer function of a linear system; the formal proof of this deficiency by Norman & Gallistel (1978) is quite recent. If we enlarge the set of stimuli to include exponentially increasing and exponentially decaying stimuli, then Norman & Gallistel provide a procedure for obtaining certain kinds of transfer functions. However, their method is so complex and so highly susceptible to data imprecision that it has never yet been attempted. Thus, sine-wave stimuli, if applicable, are preferable.

On the other hand, when the general shape of a linear system's impulse response is already known, the critical duration can provide a useful characterization of this shape and of the parametric changes that occur with factors such as adaptation level, spectral composition, area of test flash and of background field, and so on. For example, suppose the impulse response $f(t)$ is nonnegative everywhere, and suppose that the criterion response property is a peak response of at least ϵ, $\epsilon > 0$. Then critical duration $t_c = A/\max[f(t)]$, where A is the area under $f(t)$ [although $f(t)$ is a totally deterministic impulse response, whenever $f(t) \geq 0$ for all $t \geq 0$, we can characterize its shape by the parameters used to describe probability density functions]. Let the variance of $f(t)$ be σ^2. Then t_c is directly proportional to σ (the standard deviation), a common statistic used to describe the "width" of $f(t)$. For example, when $f(t) = \sigma(2\pi)^{-.5} \exp[.5(x - \mu)^2/\sigma^2]$, a normal function [of course, in the time domain, $f(t)$ can never be exactly normal], $t_c = \sigma(2\pi)^{.5}$. When $f(t) = \tau^{-n}t^{n-1}e^{-t/\tau}/(n-1)!$, a gamma function (which represents the impulse response of a series of n consecutive RC-stages, each with time-constant $RC = \tau$), $t_c = \tau(n-1)!(n-1)^{1-n}e^{n-1}$ and $\sigma = \tau\sqrt{n}$. For the single RC-stage ($n = 1$), $t_c = \tau$; that is, the critical duration *equals* the time constant.

For nonlinear systems there has been so little success in any analysis that it would be premature to judge the utility of strength-duration methods.

Supersummation:an example. Supersummation is a term used by W & K to refer to the following strength-duration phenomenon: there exist T_1 and T_2, $T_1 < T_2$ such that $E(T_1) > E(T_2)$. This means that there is a long-duration pulse (I_2, T_2) that actually requires less energy than a short-duration pulse (I_1, T_1) to produce the criterion response property R; that is, $I_2T_2 < I_1T_1$. This increased efficiency of some long pulses relative to some brief ones is the opposite of what is typically observed in strength-duration experiments. In the typical case the rule of all (I, T) pulses is: the longer the pulse, the more energy it needs to produce R. The phenomenon of supersummation is so called because brief pulses are assumed to exhibit perfect summation – a long pulse that has lower energy at its threshold than a brief pulse exhibits supersummation.

One can trivially produce supersummation on demand. Consider the following Gedanken experiment. Stimuli consist of one page of text (about 300 words) illuminated by an (I, T) pulse. The criterion response property is a subject's ability to answer correctly 70% of the questions on a subsequently administered reading-comprehension test. Let us suppose that the subject can answer these questions when the page is adequately illuminated for a duration of one minute. Making the illumination more intense might enable the subject to read slightly faster. But no intensity is great enough to yield 70% performance with durations of less than, say, 30 seconds (Fig. 2). Not only do we have supersummation (perhaps even super supersummation), but the limit as T approaches zero of $E(T)$ does not exist; $E(T)$ appeared to be heading for infinity near $T = 30$ s when the experiment was terminated because of the hazard of intensity-induced eye damage.

The reading example is deliberately obvious. To understand a page of text, we need to make eye movements, to read individual words and comprehend individual sentences, and so on. Each of these processes can be speeded up slightly by good illumination, but only modestly. Probably more than 100 eye movements are required, and one very brief flash, no matter how intense, just won't do.

Eye movements are not essential to produce supersummation. One can easily construct stimuli that contain chess or arithmetic or counting problems that can be grasped in a single fixation (i.e., they do not require eye movements) but that require time for their solution. The essence of these examples is the intrusion of high-order mental processes that require time for their execution and are relatively indifferent to intensity.

Supersummation in a linear system. To clarify supersummation, consider a simple linear system with input $i(t)$ and output $y(t)$. To be specific, we can consider a single-stage RC circuit, a linear system whose output is an exponential decay with time constant λ^{-1} when the input is an impulse. The equation for this system is $y(t) = \int_{-\infty}^{\infty} i(\tau)e^{-\lambda(t-\tau)}d\tau$.

If we choose as the criterion response property r the achievement by $y(t)$ of a particular threshold value ϵ at some particular time ($y(t) = \epsilon$ for $t = T_\epsilon$), or the achievement of a particular peak value ($\max[y(t)] = \epsilon$), or the achievement of a particular cumulative output ($\int_0^T y(t)dt \geq \epsilon$) or any of a number of other R's, then we will observe a nearly linear strength-duration function similar to that in Figure 1B. However, if we choose as the criterion response property that the output exceed a threshold value ϵ for a specified duration D – that is, that $y(t) \geq \epsilon$ for all t in some interval $(t_1, t_1 + D)$ and $y(t) < \epsilon$ for all other t – then we will observe supersummation.

For example, suppose the decay time-constant λ^{-1} of the RC system is 1 s and we wish to observe an output response that is greater than ϵ for 100 s. A 100-s input pulse will be incredibly more efficient than an impulse input whose decay must survive for 100 s. In fact, the brief impulse will have to contain about $e^{100}/105 \approx 2.6 \times 10^{41}$ more energy than a pulse of 105-s duration. The strength-duration function is illustrated in Figure 3.

Compression and time-limitations. Figure 4 shows a block diagram representation of a system that produces the strength-duration functions described above.

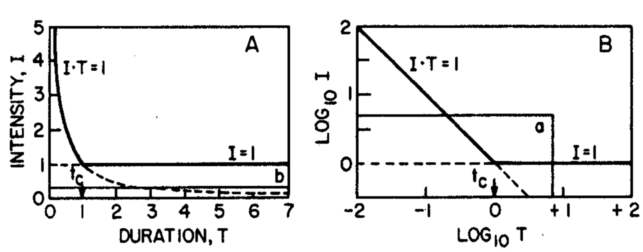

Figure 1 (Sperling). An idealized strength-duration function for (I, T) pulses of intensity I and duration T. (A) Linear coordinates; (B) Logarithmic coordinates. The critical duration as defined in text is t_c. The area of A which is represented in B is indicated by the lightly drawn box labelled a; the area of B in A is indicated by the box b.

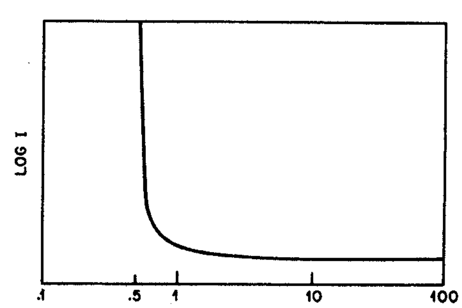

Figure 2 (Sperling). Strength-duration function for the reading Gedanken experiment. A page of text is illuminated by an (I, T) pulse; the critical response property is a score of 70% on a subsequent comprehension test.

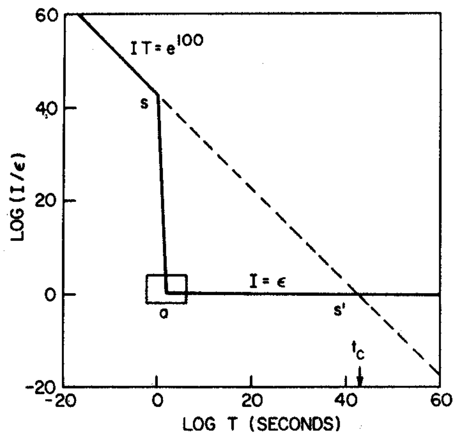

Figure 3 (Sperling). Strength-duration function for (I, T) pulse inputs to a linear, one-stage RC system with a time constant of 1 s. The criterion response property is that output remain above ϵ for 100 s. Abscissa is the log base 10 of pulse duration T; ordinate is the log base 10 of the pulse intensity I relative to ϵ. The asymptote $IT = e^{100}$ represents sensitivity only to energy; the asymptote $I = \epsilon$ represents sensitivity only to intensity; their intersection marks the critical duration t_c, which occurs at 2.7×10^{43} s or 8.5×10^{35} years. The box a indicates the ordinary range of experiments: within it, T varies from .01 s to 10^6 (11.6 days), and I varies over 10^6. The sharp corner at a seems to indicate that supersummation begins at T equal to about 100 s; however, the whole function between S and S' has supersummation relative to the $I \cdot T$ asymptote. Note similarity to Figure 2.

Figure 4 illustrates the computation of a criterion superthreshold duration being carried out by three stages in series: a threshold-transducer, a perfect integrator, and a detector. Supersummation normally occurs in systems that have a compressive intensity transformation. (A compressive transformation of x, $c(x)$ is one that has a negative second derivative – that is, $c(x)$ is concave downward. Such a transformation is called compressive because it reduces (compresses) the input's dynamic range; it is also called *saturation*.) The duration detector involves extreme compression – compression in which all intensities above ϵ are treated as equivalent. Ultimately, a system that is indifferent to intensity is concerned with other aspects of the stimulus – for (I, T) pulses the other aspect is time. Reading, counting, solving arithmetic problems, and so forth, are examples of processes that depend on time and are relatively indifferent to intensity (once legibility is achieved). Conversely, such processes may be thought of as containing extreme compressive transformations. Whether it is more helpful to consider supersummation as resulting from a compressive transformation or a time-limited process will depend on the context and on individual preference: the two ways of conceptualizing it are two sides of the same coin.

Counting dots. Hunter & Sigler (1940 *op. cit.*) presented their subjects with (I, T) pulses or arrays containing various numbers of dots. The criterion response property was a certain percentage of correct reports for each stimulus number. A complete data analysis would have to include a correction for the subject's

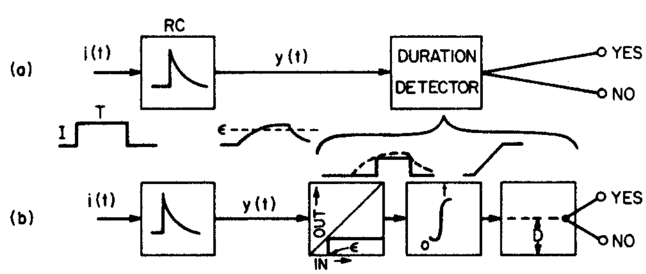

Figure 4 (Sperling). (a) Block diagram representation of a linear RC stage followed by the duration detector. The input to the system is $i(t)$; the output of the linear RC stage is $y(t)$; and the output of the whole system is either "yes" (to indicate detection) or "no" (to indicate failure to detect.) (b) is identical to (a) except that the duration-computation component has been decomposed into three components: a threshold-transducer, a perfect integrator, and a detector. Their outputs are shown. The output threshold is ϵ; the threshold duration is D.

guessing strategy and separate analyses for individual subjects. However, even from the uncorrected group data, the supersummation for numerosities of 9, 10, and 12 (the three highest tested) is so striking that it demands explanation. The explanation seems obvious: counting takes time; counting more dots takes more time. The greater the time required, the greater the relative efficiency of longer flashes, just as in the *RC* example. Unfortunately Hunter & Sigler's data are quite incomplete for high numerosities. And even if their data were better, we have already noted Norman & Gallistel's (1978) proof that strength-duration data are not, in general, rich enough to yield the transfer functions. Thus, the elucidation of dot-counting processes will require other kinds of data.

Multiple flashes. The above-mentioned reading example (a page of text illuminated by (I, T) pulse) is basically more complicated than the other examples. A sequence of brief pulses, one occurring in each fixation period, could be more efficient than any long-duration pulse with the same total energy. The (I, T) methodology would be applied to the brief pulses (the microstructure) within the pulse-train macrostructure.

In fact, even the microstructure is better studied with pairs of pulses, separated by a variable interval, than by single (I, T) pulses. A pair of pulses separated by an interval T probes the interactions (summation, inhibition, etc.) that occur at this interval far more effectively than does a single pulse of duration T (which simultaneously probes all intervals less than or equal to T). Rashbass (1970), and Broekhuijsen, Rashbass, & Veringa (1976) all provide instructive examples of the power of the method.

The two-pulse method with certain assumptions and with certain embellishments – such as permitting the pulses in pairs to be of the same or of opposite polarities – can yield a transfer function (Broekhuijsen et al. 1976). However, systems of psychological interest are seldom simple linear systems. To study nonlinear systems, one needs an even richer data base than for linear systems; for some such analyses pulse trains or mixtures of sine waves are an adequate basis (for references, see Victor & Knight 1979).

The moral. Strength-duration exoeriments can give interesting insights into systems, such as the presence of supersummation. However, the data provided by the method are inherently inadequate. Single-pulse studies must be supplemented with data from a richer assortment of stimuli [such as multiple-pulse and (multiple) sine-wave stimuli] and (if practical) a richer assortment of response measures.